Anomalous Flux-Flow Dynamics in Layered Type-II Superconductors at Low Temperatures

M. V. Feigel'man, and M. A. Skvortsov
L. D. Landau Institute for Theoretical Physics, Moscow 117940, Russia
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Low-temperature dissipation due to vortex motion in strongly anisotropic type-II superconductors with a moderate disorder ($\Delta^2/E_F \ll \hbar/\tau \ll \Delta$) is shown to be determined by the Zener-type transitions between the localized electronic states in the vortex core. Statistics of these levels is described by the random matrix ensemble of class C defined recently by Atland and Zirnbauer, so the vortex motion leads naturally to a new example of a parametric statistics of energy levels. The flux-flow conductivity σ_{xx} is a bit lower than the quasiclassical one and *grows* slowly with the increase of the electric field. [S0031-9007(97)02861-5]

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It is generally accepted that the flux-flow longitudinal (σ_{xx}) and Hall (σ_{xy}) conductivities in the mixed state of type-II superconductors are given, respectively, by the Bardeen-Steven (BS) [1] and Nozieres-Vinen [2] relations

$$\sigma_{xx} \approx \sigma_n \frac{H_{c2}}{B}; \quad \sigma_{xy} \approx (\omega_0 \tau) \sigma_{xx}, \tag{1}$$

where σ_n is the normal-state conductivity, τ is the elastic lifetime with respect to impurity scattering, and $\hbar\omega_0 \approx \Delta^2/E_F \ll \Delta$ [it is supposed in (1) that $\omega_0\tau \ll 1$]. The meaning of this relation is very simple: it reflects the fact that the core region of a vortex (of the area $\sim\xi^2$) may be considered (with respect to its electronic properties) just as a normal metal. This idea is based on the existence of the localized electronic levels within the core [3], with the energies constituting an equidistant set $E_\mu = \mu\hbar\omega_0$, where μ is a half-integer. A microscopic theory of flux-flow conductivities, which explicitly takes into account an existence of this level structure was developed long ago [4] (cf. also [5,6]). This theory is based on the quasiclassical nonequilibrium diagram technique (see, e.g., [7]); its predictions for ρ_{xx} and ρ_{xy} basically coincide with the above simple picture as far as the superconductor is not in the so-called superclean limit (i.e., if the inverse electron-impurities scattering time τ^{-1} is larger than the level separation ω_0). The underlying idea of this theory is that the interaction between electrons localized near the (moving) vortex and impurities can be treated as a kind of scattering problem in the continuous spectrum, i.e., similar to the impurity scattering in a normal state.

In the present Letter we will show that in the case of strongly anisotropic (layered) superconductors the validity of the above picture is limited to the situations when the inelastic width Γ of the core electronic levels is comparable to or larger than the level spacing ω_0. Note that $\Gamma = \Gamma_{\rm int} + \Gamma_v$, where $\Gamma_{\rm int}$ is due to electron-electron and (mainly) electron-phonon interactions and grows with temperature, whereas Γ_v is due to nonstationarity of the impurity potential around a moving vortex, and grows with its velocity v_v; therefore the condition $\Gamma \gg \omega_0$ is fulfilled at sufficiently large T or v_v. In the opposite limit, $\Gamma \ll \omega_0$, the core levels are essentially discrete; as a result, all dissipation induced by the motion of a vortex is due to rare nonadiabatic (Zener) processes of electron excitations between the core levels and subsequent downward electron transitions with emission of phonons. As a result, the longitudinal flux-flow conductivity σ_{xx} becomes reduced compared to its "quasiclassical" value (1), and dependent on the electric field:

$$\sigma_{xx} \leq \frac{ecn_e}{B}\omega_0\tau\left(1 - \sqrt{\frac{E_*}{E}}\right); \quad E_* \approx \frac{\omega_0^2 B}{c}\sqrt{\frac{\hbar}{\rho s^3}}, \tag{2}$$

where the characteristic electric field E_* is very small, so in the domain of applicability of our theory $E \gg E_*$ (here ρ is the mass density of the crystal, s is some average sound velocity, and c is the velocity of light). Note that according to Eq. (2) conductivity *grows* with the increase of electric field (or current), which is the trend opposite to the one known near the classical vortex depinning transition [8]. This mechanism of nonlinearity is different from the one predicted in [9]; the field E_* is by orders of magnitude lower than its counterpart from Ref. [9]. Our result (2) is expected to be valid in the range of not very high current densities $j \leq j_K = j_0(k_F l)^{-1/2}$, where $l = v_F\tau$ is the elastic mean free path, and $j_0 \sim en_e\Delta/p_F$ is the depairing current density. On the other hand, the current density should be higher than the critical one which is determined by pinning on the same impurity-produced potential (this point is discussed at the end of the Letter). Note that the above result refers to the moderately clean case $\omega_0 \ll \tau^{-1} \ll \Delta/\hbar$, where the *average* density of states within the core $\langle\nu_c(E)\rangle$ is constant (at $E \gtrsim \hbar\omega_0$) as in a normal metal. Therefore the effects we are discussing are of *mesoscopic nature* in the sense that they "feel" the structure of correlations of energy levels. Thus the range of existence of these effects disappears with the growth of $k_F l$: $j_K \propto (k_F l)^{-1/2}$. Mesoscopic effects of a similar origin might also exist in a superclean as well as in a dirty layered superconductor; these cases are postponed for future studies.

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We are going to consider strongly anisotropic superconductors with a high effective mass anisotropy $m_c/m_a \gg 1$; it means a very weak dispersion of the core level's energies as a function of k_z, $[E_\mu(k_z) - E_\mu(0)]/\hbar \sim \mu\omega_0 \frac{m_a}{m_c}\frac{k_z^2}{k_F^2} \ll \omega_0$. It allows us, as long as the low-lying levels with $\mu \ll m_c/m_a$ are relevant, to treat the core levels as purely discrete, which formally corresponds to a "pancake" vortex in a 2D superconductor.

At low temperatures any dissipation of energy related to the vortex motion is due to the transitions between discrete electron states within the core. In the absence of electron-phonon interaction these levels have almost zero width (the inelastic width due to electron-electron interactions for the system of discrete levels at low temperatures is even much weaker than the electron-phonon one, cf. [10]), whereas their exact locations depend on the particular realization of impurities in the region of the size $\sim\xi$ around the vortex center $\mathbf{r}_v$. The most direct way to study the statistics of random energy levels is to employ a powerful machinery of the supersymmetric sigma model [11]. Here, however, we prefer to use a technically simpler (although less general) path, sufficient for the calculation of the dissipation rate in the case $\omega_0\tau \ll 1$. This condition ensures that the position of each level E_i is strongly modified with respect to the bare levels [3] of an ideally clean superconductor (in the following we denote positive-energy levels by E_i with $i = 0, 1, \ldots$, whereas negative i will correspond to $E_i < 0$). The second important point is that the wave functions corresponding to all these levels are confined in the same area $\sim\xi^2$ within the vortex core. Therefore it is quite natural to describe the distribution of levels in terms of an appropriate random matrix ensemble (RME) [12]. When the vortex is moving, the realization of disorder in the core region is changing, so the core levels are moving up and down, presenting a new example of a "parametric level statistics" studied before with regard to disordered metallic grains and quantum dots (see, e.g., [13]). In our case the parameter $X(t)$ governing the evolution of the levels is just the vortex coordinate, so one can try to make use of the relation [13–16] between the rate of energy dissipation per pancake vortex (within the applicability of the standard Kubo approach) and the mean-squared level "velocities" $\langle (dE_i/dX)^2\rangle$:

$$(\partial W/\partial t)_{\rm Kubo} = \frac{\beta}{2}\pi\hbar C(0)\,(\partial X/\partial t)^2, \qquad (3)$$

where $\beta = 1, 2, 4$ for the system described by, respectively, orthogonal, unitary, and symplectic Wigner-Dyson ensembles, and $C(0)$ is the normalized dispersion of level velocities, $C(0) = \langle (dE_i/dX)^2\rangle/(\hbar\omega_0)^2$ (here $\hbar\omega_0 = \langle E_{i+1} - E_i\rangle$). Vortex solution breaks T invariance, whereas it leaves invariance with respect to spin rotations intact (here and below we neglect Zeeman spin splitting, which is weak as long as $B \ll H_{c2}$). Therefore one could, at first sight, conclude that the relevant for our problem Wigner-Dyson ensemble is just the unitary (U) one, with $\beta = 2$. In fact, the situation is a bit more complicated, because of the specific symmetry of the Bogolyubov–De Gennes equations determining the core levels: each of the positive-energy levels $E_i > 0$ has its exact mirror counterpart $E_{-i-1} = -E_i$. As a result, the RME relevant for our problem does not coincide exactly with U or any other of the standard Wigner-Dyson ensembles. Fortunately, the general classification of RME related to the mixed superconductive-normal systems was developed very recently by Atland and Zirnbauer (AZ) [17], who identified four different types of RME depending on the presence or absence of time- and spin-reversal symmetries. Class C of the AZ classification just corresponds to our problem at hand. The level statistics within the C class can be described via the multiparticle "wave function" of auxiliary "free fermions" in the same way as it was done for the standard unitary ensemble [14]; the only difference is the condition of vanishing amplitude of single-particle wave functions in the origin of the energy axis $E = 0$. As a result, the level statistics within the C class coincides with that of the U class as far as highly excited states with $E_i \gg \hbar\omega_0$ are involved, but differs for low-lying states.

It is very instructive to compare the dissipation rate obtained via (3) with the results of the quasiclassical kinetic equation approach [4]. As will become clear soon, the applicability of the result (3) to the vortex dynamics is limited to the case when highly excited levels are important, so at this stage we can neglect the difference between C and U ensembles, and use (3) with $\beta = 2$. Comparing the definition of the vortex damping coefficient η ($\partial W/\partial t = \eta \mathbf{v}_{\rm v}^2$) with Eq. (3), one finds

$$\pi\hbar C(0) = \eta_{\rm Kubo} = \pi\hbar n_{\rm 2D}(\omega_0\tau)\,, \qquad (4)$$

where the second equality in (4) follows from the results of [4] in the limit $\omega_0\tau \ll 1$, $n_{\rm 2D} = n_e d$ is the areal density of electrons, d is the interlayer spacing, and we put subscript "Kubo" to emphasize that the expression (4) has the same meaning and the domain of applicability as the Kubo-Greenwood formula. The damping coefficient η is proportional to the longitudinal flux-flow conductivity: $\sigma_{xx} = \eta ec/\pi\hbar B$. The parameter $C(0)$ has the dimension of inverse area; one can define a characteristic length $\mathcal{L} = C^{-1/2}(0)$ with the following meaning: when the vortex moves over the length $\mathcal{L}$, the characteristic displacements of the core levels inside it become of the order of the mean spacing ω_0. Then Eq. (4) tells us that $\mathcal{L} = (n_{\rm 2D}\omega_0\tau)^{-1/2}$. Note that $\mathcal{L}$ decreases (i.e., the sensitivity of the level positions to the shift of a vortex increases) with the decrease of disorder, and becomes of the order of the Fermi wavelength at $\omega_0\tau \sim 1$, i.e., on the border of the applicability of Eq. (4). On the other hand, in the extremely disordered limit, $k_F l \sim 1$, the length $\mathcal{L}$ would become of the order of the "clean" coherence length ξ_0.

There are two ways to understand relation (3): (i) to consider an open system with a finite width of the energy levels and use the standard Kubo-Greenwood approach [13], and (ii) to work with strictly discrete levels but take into account the nonadiabatic transitions between the time-dependent (due to variation of the external parameter X) levels $E_i(X)$, as was done in [15,16]. The first approach is clearly invalid in our case as far as the inelastic width $\Gamma \ll \omega_0$. Following the second approach [16] we find that the Kubo-like expression (4) can be safely used if the characteristic frequency of level perturbations $v_v/\mathcal{L}$ is much higher than ω_0, i.e., at $v_v \gg v_K = v_0(k_F l)^{-1/2}$, where $v_0 = \Delta/p_F$. On the other hand, at low vortex velocities $v_v \ll v_K$ and low temperatures the probability of Zener transitions between levels is exponentially low [$\sim \exp(-v_K/v_v)$] when the interlevel spacing is of the order of its average value $\hbar\omega_0$. In this case dissipation occurs when in the course of "level dynamics" the spacing between some pair of the levels becomes very small, $\delta E \leq E_Z = \hbar\omega_0(v_v/v_K)$ and Zener tunneling becomes probable.

The crucial stage of the dissipation process is determined by the nonadiabatic transitions of some electron from the highest negative-energy (filled) core level E_{-1} to the lowest positive-energy level E_0. The rate of these transitions can be calculated by the method [15] modified for the C class of the AZ classification (details will be published elsewhere [18]):

$$R_0 = v_v \int_0^\infty dA \int_0^\infty d\epsilon\, N(\epsilon, A) \exp\left(-\frac{2\pi\epsilon^2}{\hbar A v_v}\right) = \frac{\eta_{\mathrm{Kubo}}}{2\hbar\omega_0} v_v^2, \qquad (5)$$

where

$$N(\epsilon, A) = \frac{2\pi^{3/2}\epsilon}{3\hbar^3\omega_0^3}\left[\frac{A^2}{8\hbar^2\omega_0^2 C(0)}\right]^{3/2} \exp\left(-\frac{A^2}{8\hbar^2\omega_0^2 C(0)}\right)$$

is the joint probability distribution for the energy ϵ of the lowest positive level and its velocity A far from the "avoided crossing" region. Note that the rate R_0 is just the factor 2 lower than the analogous result for the U ensemble [15], which would also be valid in our problem if the transitions between highly exited levels would be considered: $R_{|i|\gg 1} \approx 2R_0$. In a similar problem treated in [15] (where all transition rates were equal, $R_i = R$), the rate of energy absorption from the source was given by $\hbar\omega_0 R$, which coincided exactly with the result for the Kubo regime, Eq. (3). This striking coincidence takes place, as was shown in [15,16], *only* for the U ensemble (ensembles with the level repulsion parameter $\beta = 1, 4$ lead to a velocity-dependent friction coefficient $\eta \propto v^{\beta/2-1}$).

In our case the situation is more complicated due to the i dependence of the transition rates R_i; as a result, the dissipation rate depends on the distribution function $f_i(t)$ for the electron's population of the ith core level, which is determined by the competition between Zener processes of excitation (the energy being absorbed from the source) and energy relaxation to the phonon "bath" caused by electron-phonon interaction. The rate of electron transitions between the core levels with energy difference $\hbar\omega$ due to the emission of 3D phonons can be estimated [18] as $\Gamma(\omega) = \omega\gamma_{\mathrm{ph}}$, where $\gamma_{\mathrm{ph}} = \nu(\omega_0\tau)\hbar\omega_0^2 n_{2D}/\rho s^3 \ll 1$ (here it is assumed that $T \leq \hbar\omega$, and ν is the numerical coefficient of order 1). Thus the kinetic equation for the distribution function $f_i(t)$ is

$$\frac{\partial f_i(t)}{\partial t} + R_{i+1}(f_i - f_{i+1}) + R_i(f_i - f_{i-1}) = -\omega_0\gamma_{\mathrm{ph}}\left(\sum_{j<i}(i - j)f_i[1 - f_j] - \sum_{j>i}(j - i)f_j[1 - f_i]\right), \qquad (6)$$

which is a kind of discrete "diffusion equation" in the energy space. Dimensional estimates show that the characteristic width of the stationary solution f_i^{st} is $i_{\mathrm{char}} \sim (R_0/\omega_0\gamma_{\mathrm{ph}})^{1/4} \sim \sqrt{v_v/v_K}\,(\Omega_D E_F/\hbar\omega_0^3\tau)^{1/4}$; usually i_{char} is large since v_v should be larger than the pinning-determined critical velocity v_c (estimate for v_c/v_K will be given below), whereas the second factor in the above estimate is always very large. One can qualitatively associate with i_{char} some "effective local temperature" $T_{eff}(v_v) \sim \hbar\omega_0 i_{\mathrm{char}} \gg \hbar\omega_0$ of the core-localized electrons. Because of the large effective width of f_i^{st}, the energy dissipation rate $\partial W/\partial t$ is close to its value $2R_0\hbar\omega_0$ for the U ensemble (all $R_i = 2R_0$), which is also the result in the Kubo regime, as mentioned above. In order to find correction $\eta - \eta_{\mathrm{Kubo}}$ we need to specify the rates R_i. It is very difficult to determine R_i for general $i \sim 1$ because of the (i) randomness of energies where avoided crossing between levels $i - 1$ and i happens and (ii) energy dependence of the mean density of states for the C ensemble. Thus we employ the simplest model with the correct asymptotic behavior of the rates: $R_{i\neq 0} = R_\infty = 2R_0$. Within this model and at $i_{\mathrm{char}} \gg 1$ we get $\partial W/\partial t = 2R_0[\hbar\omega_0 + \langle E_0\rangle(f_0 - f_{-1})]$, which leads finally to the upper bound (cf. below) for the vortex damping coefficient $\eta = (\partial W/\partial t)/v_v^2$:

$$\frac{\eta}{\eta_{\mathrm{Kubo}}} \leq 1 - \nu_1\sqrt{\frac{v_K}{v_v}}\,\gamma_{\mathrm{ph}}^{-1/4}, \qquad (7)$$

where $\nu_1 \sim 1$. Equation (7) together with the relation between vortex velocity and electric field, $v_v = cE/B$, leads to the announced in Eq. (2) result for the conductivity σ_{xx}. Equation (2) is valid up to temperatures $T \leq T_{eff}(v_v)$; at higher T the width of f_i^{st} and the damping coefficient are determined by temperature instead of vortex velocity. Although the final result for σ_{xx} is rather close to the one obtained within the quasiclassical picture (where continuous spectrum of electron states is assumed), the intrinsic mechanism of dissipation is quite different. One can understand it in the following terms: the energy dissipation rate is a product of the rate of inelastic transitions $R_{\rm in}$ by the characteristic amount of energy δE transferred at each transition; within our picture $R_{\rm in}$ is much lower than in the quasiclassic approach, whereas δE is much larger by almost the same factor. This consideration suggests that the behavior of the Hall conductivity σ_{xy}, which is of completely different physical origin, may differ considerably from the corresponding quasiclassical result $\sigma_{xy}/\sigma_{xx} \equiv \tan\theta_H \sim \omega_0\tau$. The calculation of σ_{xy} within our picture will be postponed for future studies; here we just note that it can not be done within the purely RMT approach, since the latter neglects completely the correlations between energies of the core levels and their angular momenta. Such correlations (which survive under moderate disorder) are irrelevant for the energy dissipation rate (which depends on the short-scale structure of level correlations only), but are crucial for the transverse nondissipative force acting on a vortex.

We treat Eq. (7) as an upper bound for the dissipation since we have used, when deriving Eq. (7), a model that overestimates the transition rates $R_{n\neq0}$. One could also wonder about another source of overestimation of η, due to the fact that we have employed a purely classical master equation (6) for the probability distribution function $f_i(t)$. It means that we neglected possible phase coherence of electron states on a time scale R_0^{-1} between two successive Zener transitions. The effects of localization due to a phase coherence were considered in [19] for the problem of the electron transport in mesoscopic rings. However, our case differs considerably from the one studied in [19], in that the electrons inside the moving vortex feel a time-dependent random potential which has no periodicity. We checked numerically on an appropriate model that in the absense of periodicity the diffusion over energy spectrum is retained intact even if the Zener processes are phase coherent.

In the above calculations we have assumed that the vortex is moving with a constant velocity v_v, which means that the driving current density $j = v_v e\eta/(\pi\hbar\cos\theta_H)$ is higher than the pinning-induced critical current density j_c, so the corresponding "critical velocity" $v_c \ll v_v$. On the other hand, v_v was assumed be small compared to $v_K = \Delta/\hbar k_F^{3/2} l^{1/2}$. Critical current density for the pinning of an individual pancake vortex in layered superconductor on weak impurities can be estimated [8] as $j_c \approx j_0(0.1\sigma_{\rm imp}/ld)^{1/2}$, where j_0 is the depairing current density and $\sigma_{\rm imp} = (ln_i)^{-1}$ is the electron-impurity scattering cross section (n_i is the impurity concentration). Using a "pessimistic" estimate $\theta_H \ll 1$ we get the compatibility condition for our theory in the form $5 \times 10^{-2} k_F\sigma_{\rm imp}/d \ll (\omega_0\tau)^2 \leq 1$. Thus the proposed scenario may be realized in the case of very weak impurities with a cross section small on the atomic scale. Note, however, that the above condition may actually be less stringent due to the decrease of j_c with the magnetic field already in the range $B \ll H_{c2}$ [8]. We believe that the necessary conditions for observing a nonlinear *I-V* characteristic may be fulfilled in the clean single crystals of layered superconductor $NbSe_2$ where the main source of disorder is due to random substitution of a small percentage of Nb by Ta, which is known to produce very weak impurity centers [20]. Our results may also be relevant for layered high-temperature superconductors, however, in that case the role of d-wave pairing symmetry should be studied.

In conclusion, we proposed a new mechanism of flux-flow dissipation operative in layered superconductors at low temperatures. The dissipative conductivity σ_{xx} is found to be close to the classical results [1,4] but is slowly dependent upon the electric field. We suggest that, under the same conditions, the Hall conductivity σ_{xy} may differ considerably from its quasiclassical value.

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