Dephasing in Disordered Metals with Superconductive Grains

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The temperature dependence of electron dephasing time $\tau_\varphi(T)$ is calculated for a disordered metal with a small concentration of superconductive grains. Above the macroscopic superconducting transition line, when electrons in the metal are normal, Andreev reflection from the grains leads to a nearly temperature-independent contribution to the dephasing rate. In a broad temperature range $\tau_\varphi^{-1}(T)$ strongly exceeds the prediction of the classical theory of dephasing in normal disordered conductors, whereas magnetoresistance is dominated (in two dimensions) by the Maki-Tompson correction and is positive.

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During the last few years, a number of experimental data on electron transport in disordered metal films and wires were shown to be in disagreement with the standard theory [1] of electron dephasing in normal conductors. Namely, at sufficiently low temperatures $T \leq T_1$ the dephasing rate $\tau_\varphi^{-1}(T)$ was systematically found to deviate from the power-law dependence [1]:

$$\frac{1}{\tau_\varphi^{(0)}(T)} = \begin{cases} \sim(T/\hbar)^{3/2}\tau_{\rm tr}^{1/2}/(k_F l)^2, & \text{3D case,} \\ (T/2\pi\hbar g)\ln(\pi g), & \text{2D case,} \end{cases} \quad (1)$$

with a tendency to apparent saturation in the $T \to 0$ limit ($g = \hbar/e^2 R_\square \gg 1$ is a dimensionless conductance of the film, and $\tau_{\rm tr}$ is the mean free time). Since no dephasing rate may exist at strictly zero temperature [2], such a behavior indicates the presence of some additional temperature scale(s) T_0 (which may occur to be extremely low), so that in the range $T_0 \leq T \leq T_1$ the main contribution to $\tau_\varphi^{-1}(T)$ comes from some new mechanism, different from the universal Nyquist noise considered in Ref. [1]. Among various suggestions (for a review, see Ref. [3]) there were some speculations on a possible role of electron-electron interactions in $\tau_\varphi(T)$ "saturation." Recent developments [4] of the theory [1] have proved that a *perturbative* account of electron-electron interactions does not lead to considerable corrections to Eq. (1).

In this Letter we show that electron-electron interaction considered *nonperturbatively* can indeed be responsible for strong deviation of the dephasing rate from the standard predictions. Namely, we consider a system of small superconductive (SC) islands (of size a) situated either at random or in a regular array in the bulk disordered metal matrix (3D case) or on the thin metal film (2D). Islands are supposed to be large enough to prevent suppression of superconductivity by the proximity effect; see below. Such a system can exhibit [5] a macroscopic superconducting transition mediated by the proximity Josephson coupling between the islands [6], with the transition temperature $T_c(n_i)$ depending on the concentration of the islands n_i. Above this transition electrons in the metal are normal, but Andreev reflection of them from the SC islands leads to an additional contribution to the dephasing rate:

$$1/\tau_\varphi(T) = 1/\tau_\varphi^{(0)}(T) + 1/\tau_\varphi^{\rm A}(T). \quad (2)$$

Enhancement of dephasing rate due to SC fluctuations in *homogeneous* systems was considered previously both experimentally [7] and theoretically [8]. Peculiarity of our result is that the superconductive contribution to the dephasing rate in *inhomogeneous* systems can be the dominant one in a broad range of temperatures above $T_c(n_i)$.

We consider the model system [5] where SC islands are connected to the metal matrix via tunnel barriers with normal-state tunnel conductances G_T (measured in units of $e^2/\hbar$). In the temperature range much below the critical temperature T_{c0} of islands, charge transport between them and the metal occurs due to Andreev reflection processes. We assume large Andreev conductance, $G_A \gg 1$, thus Coulomb blocking of Andreev transport is suppressed. For small concentration of the islands, $n_i < n_c \sim \exp(-\pi G_A/4)$, quantum fluctuations destroy macroscopic SC coherence through the whole system even at $T = 0$ [5,9]. In the opposite limit, $n_i \gg n_c$, the thermally driven superconductor–metal transition takes place at $T_c(n_i) \sim \hbar D n_i^{2/d}$, where D is the diffusion coefficient and d is the dimensionality of space; see Fig. 1.

Here we focus on the temperature scale $T \gg T_c(n_i)$, where macroscopic superconductivity is destroyed by thermal fluctuations, and the phases φ_j of SC order parameters on different islands fluctuate strongly and are uncorrelated with each other. Our main result is the expression for the dephasing rate due to the processes of Andreev reflection from the SC islands:

$$\frac{1}{\tau_\varphi^{\rm A}(T)} = \begin{cases} \frac{n_i}{4\nu\hbar}\left[G_A - \frac{4}{\pi}\ln\frac{G_A E_C}{2\pi^2 T}\right], & \text{3D case,} \\ \frac{n_i}{4\nu\hbar}\, G_A(T), & \text{2D case,} \end{cases} \quad (3)$$

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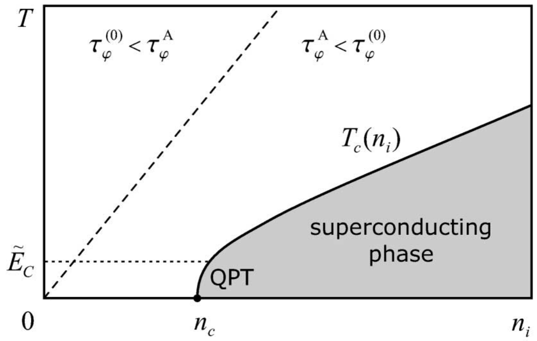

FIG. 1. Schematic (n_i, T) phase diagram of a metal with superconducting grains. The dephasing time τ_φ^A due to Andreev reflection is shorter than $\tau_\varphi^{(0)}$ in a broad range above $T_c(n_i)$.

where

$$G_A(\omega) = G_T^2/G_D(\omega) \tag{4}$$

is the (frequency-dependent) Andreev conductance of the island in the lowest tunneling approximation [10], with $G_D^{-1} = (e^2/\hbar)(4\pi\sigma a)^{-1}$ for 3D spherical islands of radii a, and $G_D^{-1}(\omega) = (4\pi g)^{-1}\ln(D/a^2\omega)$ for 2D islands of radii a. Here σ is the 3D conductivity of the metal matrix, $E_C = 2e^2/C$ is the bare island's charging energy, with C being the junction's capacitance, and ν is the metal density of states per one spin projection.

Equation (3) is valid for $T \gg \max(T_c(n_i), \tilde{E}_C)$, where $\tilde{E}_C \propto E_C e^{-\pi G_A/4}$ is the renormalized charging energy (see below). In this temperature range the dephasing rate (3) is nearly temperature independent, thus exceeding the result (1) for sufficiently small $T < T_*(n_i) \sim G_A^{2/d}(T)\, T_c(n_i)$. The window where Andreev reflection off the islands is the dominating dephasing mechanism is wide if $G_A(T) \gg 1$. In the limit of very low temperatures, $T \ll \tilde{E}_C$, accessible at $n_i < n_c$, where macroscopic superconductivity never occurs due to quantum fluctuations, the dephasing rate $1/\tau_\varphi^A \propto T$ and vanishes at $T \to 0$ in accordance with the general statement [2].

Below we provide a brief derivation of the result (3) and then discuss its physical origin and implications for observable $\tau_\varphi(T)$ in 3D and 2D systems.

Description of the formalism.—We start from the imaginary-time σ-model action functional $S = S_D + S_T$ for the disordered metal (S_D) and tunnel junctions with SC islands (S_T) [11,12]:

$$S_D = \frac{\pi\nu}{8}\mathrm{Tr}[D(\nabla Q)^2 - 4\tau_3 E Q], \tag{5}$$

$$S_T = -\frac{\pi G_T}{8}\sum_j \int \frac{dA_j}{A_j}\mathrm{Tr}Q(\mathbf{r}_j)Q_{Sj}. \tag{6}$$

Integration in Eq. (6) goes over the contact areas A_j. The space- and time-dependent matrix $Q(\mathbf{r}, \tau, \tau')$ describing electron dynamics in the metal acts in the direct product of the spin space (Pauli matrices σ_i), particle-hole (PH) space (Pauli matrices τ_i), and replica space. It satisfies the constraints $Q^2 = \mathbb{1}$ and $Q = \tau_2 Q^T \tau_2$. The usual Green functions of disordered metal correspond to the stationary uniform saddle-point $\Lambda(E_m, E_n) = \delta_{mn}\mathrm{sign}(E_m)\,\tau_3$ of the action S_D [written in the energy representation, with $E_m = \pi T(2m+1)$]. Equation (6) contains the SC matrix Q_{Sj} of the jth island: $Q_{Sj}(\tau) = \sigma_2[\tau_1\cos\varphi_j(\tau) - \tau_2\sin\varphi_j(\tau)]$. Diffusion modes of the disordered metal are described by the Q-matrix fluctuations near the saddle-point Λ, parametrized in the standard way in terms of the anti-Hermitian matrix W obeying $\{\Lambda, W\} = 0$.

Cooperon self-energy.—In the presence of SC islands cooperons acquire a gap. To obtain the cooperon self-energy due to Andreev reflection we calculate the correction to the action in the lowest tunnel approximation:

$$\delta S = -\frac{\langle S_T^{(2)} S_T^{(2)}\rangle}{2} - \langle S_T^{(3)} S_T^{(1)}\rangle + \frac{\langle S_D^{(4)} S_T^{(1)} S_T^{(1)}\rangle}{2}, \tag{7}$$

where the vertices $S_D^{(l)}$ and $S_T^{(l)}$ come from expansion of the actions (5) and (6), respectively, to the order W^l. Quadratic in G_T approximation (7) holds for $G_T \ll G_D$ [10,13]. The corresponding diagrams are shown in Fig. 2. Averaging in Eq. (7) goes over phase $\varphi_j(\tau)$ dynamics and bare diffusive modes of the matrix Q. It is important that at $T \gg T_c$ the phases on different islands are uncorrelated. The cooperon part of the induced action (7) determines the cooperon self-energy Σ_{mn}.

In the long-wavelength limit ($q \ll n_i^{1/d}$) the cooperon has the form $C(\mathbf{q}, m, n) = (Dq^2 + |\varepsilon_m + \varepsilon_n| - \Sigma_{mn})^{-1}$. Next we perform the analytic continuation to real frequencies. The quantum correction to static conductivity is controlled by the cooperon decay rate $\gamma(\varepsilon) = -\Sigma(\varepsilon, -\varepsilon)/\hbar$ [14]:

$$\gamma(\varepsilon) = \frac{n_i G_A}{2\hbar\nu}\, T\coth\frac{\varepsilon}{2T}\int_0^\infty \Pi(t)\frac{\sin(\varepsilon t/\hbar)}{\sinh(\pi T t/\hbar)}\,dt, \tag{8}$$

where $\Pi(t) = \langle\cos[\varphi(t) - \varphi(0)]\rangle$ is the real-time autocorrelation function of the island's order parameter.

The functional form of Eq. (8) coincides exactly with that for the tunneling density of states in the presence of

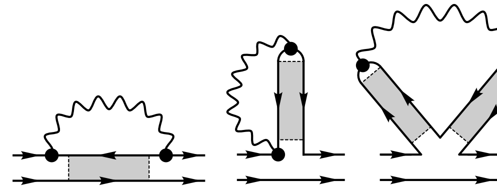

FIG. 2. Diagrams for the cooperon self-energy in the second order in G_T. Shadowed blocks are cooperons and diffusons, dots denote Andreev reflections from the island, and wavy lines stand for the phase correlation function $\Pi(\tau)$.

the Coulomb zero-bias anomaly (ZBA), cf. Eq. (58) of Ref. [15]. In the present case the island's phase $\varphi(t)$ plays the role of the Coulomb-induced phase $K(t)$ [15], whose fluctuations give rise to the ZBA. Then expression (8) can be rationalized with simple physical interpretation: SC contribution to the cooperon decay rate is just the average rate of Andreev processes which occur in the system. Indeed, quantum correction to conductivity comes from interference between different trajectories of the same electron; Andreev reflection transforms this electron into a hole, destroying further interference.

Dynamics of the phase.—Knowlegde of the phase correlation function $\Pi(t)$ is crucial for the evaluation of the integral (8). Its imaginary-time version $\Pi_M(\tau) = \langle e^{i\varphi(\tau)-i\varphi(0)}\rangle$ is determined by the single-island action [5]:

$$S_A = T\sum_k \left[\frac{\omega_k^2|\varphi_k|^2}{4E_C} + \frac{|\omega_k|G_A(\omega_k)}{8}(e^{i\varphi})_k(e^{-i\varphi})_{-k}\right], \quad (9)$$

where $\omega_k = 2\pi Tk$, $G_A(\omega)$ is given by Eq. (4), and $(e^{i\varphi})_k$ denotes the Matsubara Fourier component of $e^{i\varphi(\tau)}$.

The action (9) had been studied extensively starting from the pioneering paper [16]. The correlator $\Pi_M(\tau)$ decays at the time scale $\hbar/\tilde{E}_C$, where $\tilde{E}_C$ is the renormalized effective charging energy. For ω-independent $G_A(\omega)$ (corresponding to the 3D situation), $\tilde{E}_C$ is given by [17]

$$\tilde{E}_C \approx \frac{E_C}{3\pi^2}\left(\frac{\pi G_A}{2}\right)^4 \exp\left(-\frac{\pi G_A}{4}\right). \quad (10)$$

At $T \gg \tilde{E}_C$ the deviation of the function $\Pi_M(\tau)$ from 1 can be determined by means of the renormalization group; in the one-loop approximation [valid at $\Pi_M(\tau) \gg 1/G_A$] the result is [5]:

$$\Pi_M(\tau) = 1 - \frac{4}{\pi G_A}\ln\left(\frac{G_A E_C}{2\pi^2\hbar}\tau\right). \quad (11)$$

In the 2D case, $G_A(\omega) \propto \ln\omega$ which leads to an extremely slow ($\ln\ln\tau$) correction to $\Pi_M(\tau)$ and, hence, to negligibly small $\tilde{E}_C$ [5]. To find $\tilde{E}_C$ one then should go beyond the lowest tunneling approximation [9]. Below we will assume (for the 2D case) that temperatures are not too low and approximation (4) is valid.

Phase transition.—The temperature $T_c(n_i)$ of the *thermal* superconducting transition is determined by the mean-field relation [5]

$$T_c = \mathcal{J}(T_c)/2, \qquad \mathcal{J}(T) = \sum_i E_J(r_i, T), \quad (12)$$

where $E_J(r, T)$ is the (T-dependent) energy of proximity-induced Josephson coupling between two SC islands at the distance r in d dimensions [6]. Equation (12) is valid if the number of relevant terms in the sum for $\mathcal{J}(T_c)$ is large, otherwise the transition is not of the mean-field type, but Eq. (12) can still serve as an estimate for T_c.

The nature of the transition in d dimensions is determined by the parameter δ_d:

$$\delta_3 = \frac{3\pi^2 G_T^2}{4(k_F l)(k_F b)}, \qquad \delta_2 = \frac{G_T^2}{4g}, \quad (13)$$

which is an estimate for $E_J(b, T)/T$ at $T = \hbar D/2\pi b^2$, and $b = n_i^{-1/d}$ is the interisland distance. In three dimensions the parameter δ_3 can be arbitrary compared to 1. In two dimensions the parameter $\delta_2 \gg 1$ in the regime of weak Coulomb blockade (otherwise the transition is driven by quantum fluctuations and occurs at $E_C \sim \mathcal{J}$). If $\delta_d \ll 1$ then $T_c \ll D/2\pi b^2$, the Josephson coupling is long range and the mean-field Eq. (12) gives [14]:

$$T_c = \frac{G_T^2 n_i}{16\nu}\ln\frac{1}{\delta_d} = \frac{\hbar D}{2\pi b^2}\,\pi\delta_d\ln\frac{1}{\delta_d}, \qquad \delta_d \ll 1. \quad (14)$$

If $\delta_d \gg 1$ then $T_c \gg \hbar D/2\pi b^2$, the Josephson coupling is short range, and T_c can be estimated as

$$T_c = \frac{\hbar D n_i^{2/d}}{2\pi}\ln^2\delta_d = \frac{\hbar D}{2\pi b^2}\,\ln^2\delta_d, \qquad \delta_d \gg 1. \quad (15)$$

Quantum transition can be described within the lowest tunneling approximation only in three dimensions (see Ref. [9] for discussion of the more complicated 2D case). The point of the quantum transition is determined by the equation similar to (12): $\tilde{E}_C \simeq \mathcal{J}(0)$ [5]. The zero-temperature value of the integrated Josephson proximity coupling is given by $\mathcal{J}(0) = G_T^2 n_i/16\nu\lambda_n$, where λ_n is the Cooper-channel repulsion constant in the metal [6], and the critical concentration $n_c \simeq 16\pi\nu\lambda_n\tilde{E}_C/G_T^2$.

Dephasing rate.—To evaluate the islands' contribution into the dephasing rate, we need $\gamma(\epsilon \approx T)$. In the region of thermal fluctuations, at $T \geq \tilde{E}_C$, the integral in Eq. (8) converges at $t \sim \hbar/T$ where $\Pi(t) \approx 1$. As a result, $\gamma(\varepsilon)$ is nearly energy independent at $\varepsilon \sim T$ and can be identified with the dephasing rate leading to the result (3). The latter is valid as long as the expression in the brackets is large compared to unity. In the 3D case we kept the one-loop correction to $\Pi(t)$ given by Eq. (11). In the 2D case it is proportional to $\propto \ln\ln T$ and can be neglected compared to the bare $\ln T$ dependence of G_A.

Taking $\tau_\varphi^{(0)}(T)$ from Eq. (1), we can estimate the upper boundary $T_*(n_i)$ of the temperature range where $1/\tau_\varphi^A$ is the main contribution to the dephasing rate:

$$T_*^{3D}(n_i) \simeq 10\pi\hbar D n_i^{2/3} G_A^{2/3}, \quad (16)$$

$$T_*^{2D}(n_i) = \pi\hbar D n_i \frac{G_A(T_*^{2D})}{\ln(\pi g)}. \quad (17)$$

From the low-temperature side applicability of the result (3) is limited by the thermal transition temperature $T_c(n_i)$. Thus the relative width of the temperature window where Andreev reflection from the SC islands is the leading mechanism of dephasing is given by the ratios

$$\frac{T_*^{3D}(n_i)}{T_c^{3D}(n_i)} \approx \begin{cases} \frac{500G_A^{3/2}}{\ln^2(n_i/n_0)}, & n_i \gg n_0, \\ \frac{50G_A^{3/2}(n_0/n_i)^{1/3}}{\ln(n_0/n_i)}, & n_i \ll n_0, \end{cases} \tag{18}$$

$$\frac{T_*^{2D}(n_i)}{T_c^{2D}(n_i)} \approx \frac{20G_A(T_*)}{\ln(\pi g)\ln^2(G_T^2/4g)}, \tag{19}$$

where we used Eqs. (13)–(15), and defined $n_0 = (8\nu\hbar D/G_T^2)^3$ such that $\delta_3 = (n_i/n_0)^{1/3}$. Thus, the condition $G_A \gg 1$ guarantees the existence of the broad temperature range where the dephasing time is nearly temperature independent and given by τ_φ^A.

The region of *very low* temperatures, $T \ll \tilde{E}_C$, can be traced only in the 3D case and at very small concentration of the island, $n_i < n_c$, where superconductivity is absent even at $T = 0$ due to quantum fluctuations. Here the integral (8) converges at $t \sim \hbar/\tilde{E}_C$ yielding $1/\tau_\varphi^A(T) \sim (n_i/2\pi\hbar\nu)(T/\tilde{E}_C)$. Since $1/\tau_\varphi^A$ scales $\propto T$ it always dominates the standard 3D result (1) at $T \to 0$.

Discussion.—Experimentally, τ_φ is determined from the magnetoresistance data. For 2D systems, the low-field magnetoresistance is governed by the weak localization (WL) and Maki-Tompson (MT) corrections which have the same dependence on the magnetic field [18]:

$$\frac{\Delta R(H)}{R^2} = -\frac{e^2}{2\pi^2\hbar}[\alpha - \beta(T)]Y\left(\frac{4DeH\tau_\varphi}{\hbar c}\right), \tag{20}$$

where $Y(x) = \ln(x) + \psi(1/2 + 1/x)$ and ψ is the digamma function. Here $\alpha = 1$ $(-1/2)$ is the WL contribution in the limit of weak (strong) spin-orbit interaction, while the MT contribution is characterized by the function $\beta(T)$ expressed through the Cooper-channel interaction amplitude $\Gamma(\omega_k)$ [18]. In our system, effective attraction in the Cooper-channel emerges as a result of Andreev reflection from the SC islands. Integrating out the phases $\varphi_j(\tau)$ of the islands we obtain the standard Cooper interaction term in the action with $\Gamma(\omega_k) = (n_i G_T^2/16\nu)\Pi(\omega_k)$, which leads to $\beta(T) = (\pi^2/64)(n_i G_T^2/\nu T)$, valid at $T \gg T_c$. Comparing with the estimate (17) one finds that $\beta(T) \gg 1$ at $T \ll T_*$, that is magnetoresistance is mainly due to the MT term and thus is *positive* irrespective of the strength of the spin-orbit scattering. In the 3D case the MT correction can be either large or small compared to the WL correction, depending on temperature and other parameters of the problem.

It was implicitly assumed above that $L_\varphi \equiv \sqrt{D\tau_\varphi}$ is much longer than the interisland separation b. In the 2D case this condition is satisfied in the tunnel limit $(G_T/G_D \ll 1)$; for the 3D case the condition $L_\varphi \gg b$ is less restrictive. We expect that in the 2D case with SC islands *strongly* $(G_T \gg G_D)$ coupled to the film, $1/\tau_\varphi^A$ can be estimated analogously to Eq. (3), with the proper expression $G_A \approx G_D$ for the Andreev conductance, leading to $1/\tau_\varphi^A \sim n_i D/\ln(\xi_T/a)$, where $\xi_T = \sqrt{\hbar D/2\pi T}$. Finally, suppression of superconductivity in the islands can be neglected as long as their SC gap $\Delta \gg G_T/(\nu V)$, with V being the island's volume [9].

Taking, for example, the 2D film with the sheet resistance $R_\square = 600\ \Omega$, $l = 0.6$ nm, $k_F = 10\ \text{nm}^{-1}$, $D = 3\ \text{cm}^2/\text{s}$, and islands of radius $a = 40$ nm situated at the distance $b = 300$ nm, we find that our theory holds for $5 < G_T < 20$. Choosing $G_T = 10$ (corresponding to the transparency per channel $\Gamma \sim 10^{-3}$), we have the experimentally accessible crossover temperature $T_* \sim 0.1$ K.

To conclude, we demonstrated that a small concentration of superconductive islands can enhance considerably the low-temperature dephasing rate in disordered metals as seen via the low-field magnetoresistance.

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