Quantum Superconductor-Metal Transition in a Proximity Array

M. V. Feigel'man,[1] A. I. Larkin,[1,2] and M. A. Skvortsov[1]

[1]*L. D. Landau Institute for Theoretical Physics, Moscow 117940, Russia*

[2]*Theoretical Physics Institute, University of Minnesota, Minneapolis, Minnesota 55455*

(Received 12 October 2000)

A theory of the zero-temperature superconductor-metal transition is developed for an array of superconductive islands (of size d) coupled via a disordered two-dimensional conductor with the dimensionless conductance $g = \hbar/e^2R_\square \gg 1$. At $T = 0$ the macroscopically superconductive state of the array with lattice spacing $b \gg d$ is destroyed at $g < g_c \approx 0.1\ln^2(b/d)$. At high temperatures the normal-state resistance between neighboring islands at $b = b_c$ is much smaller than $R_Q = h/4e^2$.

DOI: 10.1103/PhysRevLett.86.1869 PACS numbers: 74.40.+k, 71.30.+h, 74.50.+r

In two-dimensional (2D) systems, qualitative arguments based on duality between Cooper pairs and vortices lead to the prediction [1] that the superconductor-insulator transition happens at the universal quantum value $R_Q \equiv h/4e^2$ of the resistance per square $R_\square$. Although a number of experiments (cf. [2] and references therein) seem to be in agreement with this prediction, other data demonstrate strong deviations from it [3–6]. A phenomenological picture of duality is not able to predict the system's parameters (e.g., the value of normal-state resistance) leading to the quantum critical point: a microscopic theory is needed to find it. Competition between Josephson coupling E_J and charging energy E_C is known [7,8] to be the driving mechanism of zero-temperature phase transitions between the superconductive and insulating states in artificial arrays [3,9], films [10,11], and bulk materials [12]. In such systems there are no free electrons at very low temperatures due to Cooper pairing, but pairs may become localized due to Coulomb repulsion. This is the "bosonic" mechanism of superconductivity suppression. Homogeneously disordered superconductive films [4–6,13] present another group of systems where quantum fluctuations lead to destruction of superconductivity. The theory of T_c suppression in such films was developed in Ref. [14]. The qualitative idea behind this theory is that disorder-enhanced Coulomb repulsion leads to the decrease of Cooper attraction and thus to the decrease of T_c. The superconductive transition temperature vanishes [14,15] when the dimensionless film conductance $g = \hbar/e^2R_\square$ decreases down to $g_{\mathrm{Fin}} = (2\pi)^{-2}\ln^2(1/T_{c0}\tau_{\mathrm{tr}})$, where T_{c0} is the BCS transition temperature and τ_{tr} is the elastic scattering time. This second ("fermionic") mechanism of superconductivity suppression is clearly different from the first one [7] since its basic feature lies in the disappearance of Cooper pairs altogether. Experimental data supporting the fermionic mechanism are reviewed in Ref. [15]. A drawback of this theory is that it neglects quantum fluctuations of the bosonic field (i.e., it can be considered as a kind of the BCS theory with the renormalized attraction constant). For a phenomenological comparison of the bosonic and fermionic mechanisms, cf. Ref. [16].

In this Letter we study a model for quantum breakdown of superconductivity, which lies in between the two limiting cases discussed above. We consider an array of small superconductive (SC) islands (of radius d each) in contact with a thin film of dirty normal (N) conductor with dimensionless conductance $g \gg 1$. The distance between neighboring islands is $b \gg d$. Resistance R_T of the interface between each island and the film is low: $G_T = \hbar/e^2R_T \gg 1$. Islands are thick enough to prevent suppression of superconductivity inside them; the corresponding condition for the superconductive gap reads $\Delta_{\mathrm{SC}} \gg G_T/\nu V_i$, where V_i is the island's volume and ν is the density of states. We assume also that $G_T^2 \gg 4\pi g$; the reason for that is explained below. We show that macroscopic superconductivity in such a system at $T = 0$ becomes unstable with respect to quantum fluctuations at g less than

$$g_c = \mathcal{G}_c\left(\frac{1}{\pi}\ln\frac{b}{d}\right)^2, \tag{1}$$

where $\mathcal{G}_c \sim 1$ will be determined below. Equation (1) presents our main result (obtained within logarithmic accuracy), which shows that the critical sheet resistance $R_{\square c} = \hbar/e^2g_c$ is much less than the quantum resistance R_Q, provided $\ln(b/d) \geq 3$. Moreover, the same is valid for the normal-state resistance between neighboring islands $R_n = (R_{\square c}/\pi)\ln(b/d) \approx 3R_Q/\ln(b/d)$. This result is at odds with the usual arguments based on the model of resistively shunted Josephson junctions [17], which predict superconductive behavior of a single junction to be preserved at $T = 0$ if $R_n < R_Q$. This discrepancy is due to the discrete nature of charge transport between SC islands (neglected in the model [17]).

We follow an idea presented in Ref. [18], where a simplified version of the considered model was analyzed (cf. also [19]). Namely, we make use of the long-range nature of the Josephson coupling J_{ij} between SC islands due to the proximity effect in the film, which scales as $J_{ij} \propto r_{ij}^{-2}\exp[-c(r_{ij}/L_T)]$, where $L_T = \sqrt{\hbar D/T}$ is the thermal coherence length (cf. Refs. [18,20]), $c \sim 1$, and

D is the diffusion constant in the film. At low temperatures the interaction radius L_T diverges indicating that the position of the quantum phase transition can be found in the mean-field approximation (MFA) analogous to the one developed in Ref. [21]. Within the MFA, macroscopic superconductive coherence sets in at

$$\frac{1}{2} J(T)C(T) \geq 1, \quad \text{where } C(T) = \int_0^{1/T} d\tau\, C_0(\tau), \tag{2}$$

where $J(T) = \sum_j J_{ij}$, and $C_0(t) = \langle \cos[\theta(0) - \theta(t)] \rangle$ is the single-island autocorrelation function of the order parameter phase. At low interface transparencies, $G_T \ll 4\pi g \lambda_n$, one obtains [18,20] $J(0) = (G_T^2/16\nu\lambda_n^2)[1/b^2 \ln(b/d)]$, where λ_n is the Cooper repulsion constant at the energy scale $\omega_d = D/d^2$. In the opposite limit of large G_T we can neglect λ_n and obtain $J(0)$ solving the Usadel equation at $\ln(b/d) \gg 1$:

$$J(0) = (\pi^4/2)gD/b^2 \ln(b/d). \tag{3}$$

The key point in the discussion of the $T = 0$ transition is to determine $C(T \to 0)$. We will see that $C(0)$ depends exponentially on the film conductance g. If islands do not have Ohmic contacts with the film (coupling via capacitance C_j only) then $C(0) = \hbar/E_C = \hbar C_j/2e^2$. In our case $\hbar/C(0)$ plays the role of an effective charging energy E_C^* of an island that survives in spite of a good conductance around. To make ideas transparent, we first discuss a simplified model [18] with sufficiently strong Cooper-channel repulsion in the film, $\lambda_n \gg G_T/4\pi g$. Then dynamics of the phase $\theta(t)$ of a single SC island can be described by a simple imaginary-time action,

$$S_0[\theta] = -\frac{G_A}{8\pi} \int_0^{1/T} dt\, dt' \, \frac{\cos[\theta(t) - \theta(t')]}{(t - t')^2}. \tag{4}$$

Here $G_A = G_T^2/4\pi g \lambda_n$ is the Andreev subgap conductance (normalized to $e^2/\hbar$) in the limit of the weak proximity effect, valid under the condition $\lambda_n \gg G_T/4\pi g$ [18] (that paper contains a numerical mistake, corrected in [22]). Expression (4) is valid at low energies $\omega \leq \omega_d e^{-1/\lambda_n}$, at higher $\omega \leq \omega_d$ one has $G_A(\omega) = (G_T^2/4\pi g)\ln(\omega_d/\omega)$. The above-mentioned condition $G_T^2 \gg 4\pi g$ is necessary to have large conductance for all $\omega \leq \omega_d$, in order to neglect trivial Coulomb blockade effects. For large G_A one can start from the Gaussian approximation for $S_0[\theta(t)]$. Then the Fourier-transformed correlator of phase fluctuations $\langle |\theta_\omega|^2 \rangle_0 = 4/|\omega| G_A$, and, hence, $C_0(t) = e^{-\frac{1}{2}\langle [\theta(t) - \theta(0)]^2 \rangle_0} \propto t^{-4/\pi G_A}$. At $G_A > 4/\pi$, $C(T \to 0)$ diverges which seems to indicate that at large G_A superconductivity is always stable at $T = 0$, in agreement with [17]. The crucial point is to note that the employed Gaussian approximation breaks down at a finite time scale t^*, due to downscale renormalization of G_A. This renormalization is caused by the periodicity of the action $S_0[\theta]$ as a functional of $\theta(t)$, that is, in physical terms, by charge quantization. This problem is analogous to the one studied by Kosterlitz [23]. Translating his results to the present case, one gets the renormalization group (RG) equation $dG_A(\zeta)/d\zeta = -4/\pi$, with $\zeta = \ln \omega_d t$. This equation is to be solved with the initial condition $G_A(0) = G_A$. As a result, at the time scale $t^* \sim \omega_d^{-1} e^{\pi G_A/4}$ the renormalized Andreev conductance $G_A(t^*)$ decays down to the value of order unity [18]. At longer time scales $C_0(t)$ decays approximately as t^{-2}, so the integral $C(0) \sim t^* \sim \omega_d^{-1} e^{\pi G_A/4}$. Taking into account that $J \sim b^{-2}$, and using Eq. (2), one obtains [18] the critical distance between islands $b_c \sim d e^{\pi G_A/8}$.

However, this result is valid under the condition $\lambda_n \gg G_T/4\pi g$ which is difficult to realize simultaneously with the inequality $G_T^2 \gg 4\pi g$. Indeed, at energies $E \ll \hbar/\tau_{\rm tr}$, Cooper interaction constant $\lambda(E)$ is determined by the RG equation [14] which we present in a simplified form [22] valid for $\ln(\hbar/E\tau_{tr}) \ll g$ when renormalization of g can be neglected:

$$\frac{d\lambda}{d\zeta} = -\lambda^2 + \lambda_g^2, \qquad \zeta = \ln \frac{\omega_d}{E}, \qquad \lambda_g = \frac{1}{2\pi\sqrt{g}}, \tag{5}$$

and $\lambda(\zeta = 0) = \lambda_n$. The fixed point solution of Eq. (5), $\lambda = \lambda_g$, is too small to fulfill both of the above inequalities together. Therefore, typically the approximation of single-parameter RG for G_A is not valid, and we should reconsider the problem of the subgap N-S conductance in the presence of three different effects acting simultaneously: (i) disorder-enhanced multiple Andreev reflections [24]; (ii) Cooper-channel repulsion λ which reduces G_A [18,25,26]; (iii) quantum fluctuations of the phase $\theta(\tau)$ which destroy coherence between Andreev reflections and suppress $G_A(\zeta)$ at long time scales. To treat all these effects, we employ the functional RG method for the proximity-effect action in the Keldysh form [26].

As in the simplified model [18] discussed above, the constant $C(0)$ is determined (with exponential accuracy) by the value of time t^*, when $G_A(\zeta = \ln \omega_d t^*)$ becomes of the order of 1, since at longer times $C_0(t)$ decays fast. However, the equation for $G_A(\zeta)$ is much more complicated now as it includes an infinite set of parameters. To derive the corresponding RG equations, we start from the Keldysh action for a SC island in contact with a disordered metal, derived in Ref. [22]. It can be represented as a sum $S = S_{\rm bulk} + S_{\rm bound}$ of the bulk and boundary [the last term in Eq. (6)] contributions:

$$S = \frac{i\pi\nu}{4} \mathrm{Tr}[D(\nabla Q)^2 + 4i(i\tau_z \partial_t + \overleftrightarrow{\phi} + \overleftrightarrow{\Delta})Q] + \mathrm{Tr}\vec{\phi}^T V^{-1} \vec{\phi} + \frac{2\nu}{\lambda} \mathrm{Tr}\vec{\Delta}^+ \sigma_x \vec{\Delta} - \frac{i\pi G_T}{4} \mathrm{Tr} Q_S Q. \tag{6}$$

The bulk action, $S_{\rm bulk}$, is a functional of three fluctuating fields: the matter field $Q(\mathbf{r}, t, t')$ in the film [its average value gives the electron Green function $G(\mathbf{r}, \mathbf{r}')$ at $\mathbf{r} = \mathbf{r}'$], the electromagnetic potential $\vec{\phi}(\mathbf{r}, t)$, and the order parameter field $\vec{\Delta}(\mathbf{r}, t)$ used to decouple the quartic interaction vertex in the Cooper channel. $Q(\mathbf{r}, t, t')$ is a matrix in the time domain, and in the direct product $K \otimes N$ of the Keldysh and Nambu-Gor'kov spaces. Pauli matrices in the K and N spaces are denoted by σ_i and τ_i, respectively. The field Q satisfies a nonlinear constraint $Q^2 = 1$ and can be parametrized as $Q = e^{-W/2}\Lambda e^{W/2}$ with $\{W, \Lambda\} = 0$, where $\Lambda = \Lambda_0\tau_z$ is the metallic saddle point and $\Lambda_0(\epsilon) = \sigma_z + 2\sigma_+ F(\epsilon)$, whereas the matrix $F(\epsilon) = \tau_0 f(\epsilon) + \tau_z f_1(\epsilon)$ has the meaning of a generalized distribution function. The object $\vec{\phi} = (\phi_1, \phi_2)^T$ is a vector in the Keldysh space, with ϕ_1, ϕ_2 being the classical and quantum components of the ϕ field. $\overleftrightarrow{\phi}$ is a shorthand notation for the matrix $\overleftrightarrow{\phi} = \phi_1\sigma_0 + \phi_2\sigma_x$. Similarly, $\vec{\Delta} = (\Delta_1, \Delta_2)^T$, and $\overleftrightarrow{\Delta}$ stands for a 4×4 matrix $\overleftrightarrow{\Delta} = [\tau_+\Delta_1 - \tau_-\Delta_1^*]\sigma_0 + [\tau_+\Delta_2 - \tau_-\Delta_2^*]\sigma_x$, where $\tau_\pm \equiv (\tau_x \pm i\tau_y)/2$. In terms of the σ-model action (6), diffuson and Cooperon collective modes of the electron system are described as slow fluctuations of the Q matrix over the manifold $Q^2 = 1$. The last (boundary) term in Eq. (6) describes an elementary tunneling process between the SC island and the N metal. The matrix Q_S describes the state of the SC island. At the low-energy scales $\epsilon \ll |\Delta_{\rm SC}|$ it is expressed via the phase $\theta(t)$: $Q_S = -i\tau_+ e^{i\overleftrightarrow{\theta}} + i\tau_- e^{-i\overleftrightarrow{\theta}}$, where $\overleftrightarrow{\theta} = \theta_1\sigma_0 + \theta_2\sigma_x$.

The action (6) contains a fluctuating scalar potential field $\vec{\phi}$ accounting the Coulomb interaction in the density channel. Its main effects are (i) local electroneutrality of the electron liquid at low frequencies and (ii) zero-bias anomaly in the tunneling density of states [27]. Both effects can be taken care of by means of a special gauge transformation [28]: $Q_{tt'} \to e^{i\vec{K}(t)\tau_z} Q_{tt'} e^{-i\vec{K}(t')\tau_z}$ and $\vec{\phi}(t) \to \vec{\phi}(t) + \partial_t \vec{K}(t)$. Choosing the "Coulomb phase" $\vec{K}(t)$ according to Ref. [28] one obtains that the effect (i) is contained in the tree level of the transformed effective action, whereas (ii) comes from the simplest loop correction [22,28]. After the above gauge transformation the phase $\vec{\theta}(t)$ enters the action in the combination $\vec{\theta}(t) - 2\vec{K}(t)$ only. Now the key point comes about: the phase $\vec{\theta}(t)$ is not fixed by any external source and should be integrated out. Thus the shift of integration variable $\vec{\theta}(t) \to \vec{\theta}(t) - 2\vec{K}(t)$ eliminates $\vec{K}(t)$ from the action, together with both effects (i) and (ii). In other terms, the present problem of unconstrained phase $\vec{\theta}(t)$ fluctuations can be treated as if it would be no Coulomb interaction, since Gaussian terms in the action containing electric field are decoupled from the redefined $\vec{\theta}(t)$ variable. It is thus legitimate to neglect electroneutrality and calculate frequency-dependent subgap conductance $G_A(\omega)$ as if the outer normal contact would be placed at the distance $R_\omega = \sqrt{D/\omega}$ from the SC island. We emphasize that the same would be wrong for a usual problem of N-S conductance between contacts with fixed voltages, where the full size of the N film, $L \gg R_\omega$, does enter the result, adding the term $(R_\square/2\pi)\ln(L/R_\omega)$ into the resistance; cf. Ref. [26].

Next we use the RG method to integrate consecutively over fast degrees of freedom in the action (6), which is defined with ω_d being a high-energy cutoff. At each step of the RG procedure one has to eliminate fast diffusons and Cooperons in the N film [22], and fast (with $\Omega^* > \omega > \Omega$) fluctuations of the order-parameter phase $\vec{\theta}$ on the SC island (where Ω is the running infrared RG cutoff). The above integration results in a correction to the action of slow variables proportional to $\Delta\zeta = \ln(\Omega^*/\Omega)$. The structure of the boundary term in the action (6) is not reproduced under the RG [22]; instead higher-order terms $\mathrm{Tr}(Q_S Q)^n$ are generated, which are all relevant in the case of the strong proximity effect. The full boundary action can be written in the form [26]

$$S_{\rm bound} = -i\pi^2 g \sum_{n=1}^{\infty} \gamma_n(\zeta)\,\mathrm{Tr}(Q_S Q)^n. \tag{7}$$

In the model [18] of large λ_n discussed above, the separation of scales was possible: at relatively short scales Cooperon modes were the only relevant ones. Under the RG the term $\gamma_2\,\mathrm{Tr}(Q_S Q)^2$ was generated [22] and led to a constant value of $G_A \gg 1$ (other $\gamma_{n\ge 3}$ were still small). At longer time scales fluctuations of θ became important, being determined by the action (4). In the full problem considered now, all parameters γ_n are important, and all types of fluctuations should be considered simultaneously. The corresponding RG equations were derived in [26] for the case when the SC island is connected to an external circuit and its phase is fixed. In the absence of an external contact, high-frequency fluctuations of the phase are given by $\langle\theta_i(\omega)\theta_j(-\omega)\rangle = 2\Pi_\omega^{ij}/\omega G_A(\zeta_\omega)$ with $\Pi_\omega = (\sigma_0 + \sigma_z)\coth(\omega/2T) + i\sigma_y$ and $\zeta_\omega = \ln(\omega_d/\omega)$ and lead to the new term in the RG equations:

$$\Delta\gamma_n = -\frac{2\Delta\zeta}{\pi G_A(\zeta)}\left(n\gamma_n + 2\sum_{k=1}^{\infty}(-1)^k(n + 2k)\gamma_{n+2k}\right). \tag{8}$$

It is convenient [26] to introduce a function of an auxiliary continuous variable x according to $u(x,\zeta) = \sum_{n=1}^{\infty} n\gamma_n(\zeta)\sin nx$. Then the full RG equation for the function $u(x,\zeta)$ reads [with the left-hand side (lhs) derived in [26] and the right-hand side (rhs) being the Fourier transform of Eq. (8)]:

$$u_\zeta + uu_x + \lambda(\zeta)u(\tfrac{\pi}{2},\zeta)\sin x = -\frac{2}{\pi G_A(\zeta)}\,\mathcal{F}[u(x,\zeta)], \tag{9}$$

where $\mathcal{F}[u(x,\zeta)] \equiv [u(x,\zeta)\tan x - u(\frac{\pi}{2},\zeta)\sec x]_x$ and the initial condition is $u(x,0) = (G_T/4\pi g)\sin x$. The scale-dependent subgap conductance $G_A(\zeta)$ is determined by the solution of Eq. (9) as $G_A(\zeta) = 4\pi g u_x(\frac{\pi}{2},\zeta)$.

To find the parameter $C(0)$ with an exponential accuracy, we integrate Eq. (9) together with Eq. (5) for $\lambda(\zeta)$. Written in the rescaled variables $s = \zeta/2\pi\sqrt{g}$, $w(x,s) = 2\pi\sqrt{g}u(x,\zeta)$, and $\tilde{\lambda} = \lambda/\lambda_g$, Eq. (9) acquires the form

$$w_s + w w_x = -2\,\frac{\mathcal{F}[w(x,\zeta)]}{w_x(\frac{\pi}{2},s)} - \tilde{\lambda}(s) w(\tfrac{\pi}{2},s)\sin x\,, \tag{10}$$

with the initial condition $w(x,0) = A\sin x$, where $A \equiv G_T/2\sqrt{g} \gg 1$. The solution of Eq. (10) weakly depends on the ratio $\tilde{\lambda}(0) = \lambda_n/\lambda_g$ which is assumed to be not very large. At $s \ll 1$, the function $w(x,s)$ is close to the solution of Eq. (10) with zero rhs, which, at $s \geq A^{-1}$, is given by $w(x,s) \approx x/s$ for $x \in (0,\pi)$. As s grows, the rhs terms become increasingly important and eventually reduce $G_A(\zeta) = 2\sqrt{g}\,w_x(\frac{\pi}{2},s)$ down to the value of the order of 1 at the critical value of $\zeta^* = 2\pi\sqrt{g}\,s_c$. Therefore, $C(0) \sim \omega_d^{-1} e^{2\pi\sqrt{g}\,s_c}$. The value of $s_c \sim 1$ was determined, for several values of $\tilde{\lambda}(0)$, via the numerical solution of Eq. (10) in the limit $A \to \infty$. Using Eqs. (2) and (3), we obtain the result (1) with $\mathcal{G}_c = s_c^{-2}$:

$$\mathcal{G}_c = \begin{cases} 0.64, & \text{for } \tilde{\lambda}(0) = 0\,; \\ 0.73, & \text{for } \tilde{\lambda}(0) = 1\,; \\ 0.79, & \text{for } \tilde{\lambda}(0) = 2\,. \end{cases} \tag{11}$$

At $g > g_c(b)$ macroscopic superconductive transition occurs at $T > 0$. Close to the critical point (1), at $b \leq b_c(g)$, the transition temperature is primarily determined by the temperature dependence of $\mathcal{J}(T) = \mathcal{J}(0)\ln(L_T/b)/\ln(L_T/d)$. The same expression, with L_T replaced by $l_H = \sqrt{\pi\hbar c/eH}$, determines $\mathcal{J}(0)$ in the presence of transverse magnetic field H. To find the critical temperature $T_c(b)$ and the critical magnetic field $H_{c2}(T=0)$, one uses Eq. (2) together with the above expressions for $\mathcal{J}$. The result is that both $T_c(g)$ and $H_{c2}(g)$ scale in the same way and drop fast at $b \to b_c(g)$:

$$\ln\frac{T^*}{T_c} \approx \ln\frac{\Phi_0}{H_{c2}b^2} \approx \frac{2\ln(b/d)}{b_c^2(g)/b^2 - 1}\,, \tag{12}$$

where $T^* = \hbar D/b^2$. Equation (12) is valid for $b/b_c(g) \geq [2\ln(b/d)]^{-1/2}$. This inequality ensures that T_c is small compared both to T^* (under this condition the proximity coupling is long range) and to $\hbar/C(0)$. The latter condition allows one to approximate $C(T)$ by $C(0)$ while deriving Eq. (12). On the other hand, our results are limited to $T \geq T_{\mathrm{loc}} \sim (\hbar/\tau_{\mathrm{tr}})e^{-\pi^2 g}$, since we neglected weak localization effects. At shorter $b \ll b_c(g)\sqrt{\lambda_g}$, the transition occurs at $T_c \sim T^*$. Here $L_{T_c} \sim b$, the MFA is not applicable, and the transition is governed by thermal fluctuations. The magnetic field $\sim\Phi_0/b^2$ drives such an array into the superconductive glass state.

Because of the long-range nature of proximity couplings, our main result (1) is robust to a moderate disorder in sizes and positions of the islands. With the choice of parameters like $d = 10$ nm, $b = 0.5$ μm, and $D = 10$ cm^2/s; one finds the critical conductance $g_c > 1$, with the characteristic temperature scale $T^* \approx 30$ mK.

In conclusion, we have developed a theory of quantum superconductive-metal transition in a 2D proximity-coupled array. This transition can be traced by continuous variation of the carrier density in the 2D film. The critical value of the *bare* film resistance $R_{\square c}$ is nonuniversal and small compared to R_Q if $\ln(b/d) \geq 3$. Under the same condition, there exists a broad temperature range $T_{\mathrm{loc}} \leq T \leq T^*$ where localization effects are weak and our results are applicable. The issue of universality [1] of the *renormalized* critical resistance $R_{\square c}(T \to 0)$ is left unresolved, since we employed the approximation of $g \gg 1$. Near the quantum critical point the system behaves as a BCS-like superconductor with the effective Cooper attraction constant vanishing at $R_\square \to R_{\square c}$.

We are grateful to A. Kamenev and Yu. V. Nazarov for useful discussions. This research was supported by NSF Grant No. DMR-9812340 (A. I. L.), RFBR Grant No. 98-02-16252, NWO-Russia collaboration grant, Swiss NSF Grant No. 7SUPJ062253.00, and by the Russian Ministry of Science.

[1] M. P. A. Fisher, Phys. Rev. Lett. **65**, 923 (1990).
[2] A. M. Goldman and N. Marcovic, Phys. Today **51**, No. 11, 39 (1998).
[3] H. S. J. van der Zant *et al.,* Phys. Rev. Lett. **69**, 2971 (1992); Phys. Rev. B **54**, 10 081 (1996).
[4] V. F. Gantmakher *et al.,* JETP Lett. **71**, 160 (2000).
[5] J. A. Chervenak and J. M. Valles, Jr., Phys. Rev. B **61**, R9245 (2000).
[6] N. Mason and A. Kapitulnik, cond-mat/0006138.
[7] K. B. Efetov, Sov. Phys. JETP **51**, 1015 (1980).
[8] R. Fazio and G. Schön, Phys. Rev. B **43**, 5307 (1991).
[9] P. Delsing *et al.,* Phys. Rev. B **50**, 3959 (1994).
[10] H. M. Jaeger *et al.,* Phys. Rev. B **40**, 182 (1989).
[11] D. B. Haviland *et al.,* Phys. Rev. Lett. **62**, 2180 (1989).
[12] V. F. Gantmakher *et al.,* JETP Lett. **64**, 767 (1996).
[13] A. F. Hebard and M. Paalanen, Phys. Rev. Lett. **65**, 927 (1990).
[14] A. M. Finkelstein, JETP Lett. **45**, 46 (1987); *Soviet Scientific Reviews,* edited by I. M. Khalatnikov (Harwood Academic, London, 1990), Vol. 14.
[15] A. M. Finkelstein, Physica (Amsterdam) **197B**, 636 (1994).
[16] A. I. Larkin, Ann. Phys. (Leipzig) **8**, 507 (1999).
[17] A. Schmid, Phys. Rev. Lett. **51**, 1506 (1983); S. A. Bulgadaev, JETP Lett. **39**, 314 (1984).
[18] M. V. Feigel'man and A. I. Larkin, Chem. Phys. **235**, 107 (1998).
[19] B. Z. Spivak *et al.,* cond-mat/0004058.
[20] L. A. Aslamazov, A. I. Larkin, and Yu. N. Ovchinnikov, Sov. Phys. JETP **28**, 171 (1969).
[21] V. G. Vaks, A. I. Larkin, and S. A. Pikin, Sov. Phys. JETP **26**, 188 (1968).
[22] M. V. Feigel'man, A. I. Larkin, and M. A. Skvortsov, Phys. Rev. B **61**, 12 361 (2000).
[23] J. M. Kosterlitz, Phys. Rev. Lett. **37**, 1577 (1976).
[24] F. W. J. Hekking and Yu. V. Nazarov, Phys. Rev. B **49**, 6847 (1994).
[25] F. Zhou *et al.,* Phys. Rev. B **52**, 4467 (1995); T. H. Stoof and Yu. V. Nazarov, Phys. Rev. B **53**, 14 496 (1996).
[26] M. A. Skvortsov, A. I. Larkin, and M. V. Feigel'man, Phys. Rev. B (to be published), cond-mat/0008463.
[27] B. L. Altshuler *et al.,* Phys. Rev. Lett. **44**, 1288 (1980).
[28] A. Kamenev and A. Andreev, Phys. Rev. B **60**, 2218 (1999).