Quantum and thermal depinning of a string from a linear defect

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The problem of a massive elastic string depinning from a linear defect under the action of a small driving force is considered. To exponential accuracy the decay rate is calculated with the help of the instanton method; then, fluctuations of the quasiclassical solution are taken into account to determine the preexponential factor. The decay rate exhibits a kind of first-order transition from quantum tunneling to thermal activation with a vanishing crossover region. The model may be applied to describe nucleation in two-dimensional first-order quantum phase transitions. [S0163-1829(97)02901-9]

I. INTRODUCTION

Macroscopic tunneling phenomena have attracted theorists' attention since it is the field where classical and quantum physics meet each other. It governs the low-temperature decay of metastable states of physical systems with many degrees of freedom, separated from the neighboring states by high potential barriers; at high temperatures the metastable state decays via thermal activation. Several such systems were studied, including motion of dislocations across Peierls barriers,[1,2] tunneling with strong friction,[3] or quantum breaking of an elastic string.[4]

In the present paper the problem of quantum and thermal depinning of a massive elastic string trapped in a linear defect and subject to a small driving force is considered. Interest in one-dimensional manifold tunneling has been revived during the last years as it describes the creep of vortices trapped by columnar defects or plane twin boundaries.[5] Unlike thermal activation, which for nondissipative Hamiltonian systems is independent of the dynamic properties of the string,[6] quantum tunneling is strongly dependent on the dynamics, which for the case of vortices in high-temperature superconductors (HTSC's) can be either dissipative or governed by a Hall force.[5] The tunneling problem of a massive string therefore cannot be directly applied to vortex creep but its study is important since it allows for an exact analytical solution. Massive string depinning from a linear defect describes the nucleation of a new phase in the vicinity of a first-order quantum phase transition in two dimensions. The boundary of the two phases may be considered as a string, the difference in chemical potentials of the phases playing the role of a driving force.

The problem is solved quasiclassically and exact solutions are obtained for the whole temperature range in the limit of a small driving force. Then, fluctuations of the quasiclassical solutions are taken into account to determine the preexponentials. The special feature of the problem considered is the presence of a "first-order" transition between the quantum tunneling and thermal activation regimes that should be contrasted with a smooth crossover between them in other related problems.[1,2,7]

II. MODEL AND SUMMARY

Consider a massive elastic string on a plane pinned by a linear defect. The dynamics of the string is governed by the Lagrangian

$$\mathcal{L}[u(x,t)]=\int dx\left[\frac{\rho}{2}\dot{u}^2-\frac{\rho c^2}{2}(\partial_x u)^2-V(u)\right]. \quad (1)$$

Here ρ is the mass density of the string, c is the speed of sound, and u is the displacement perpendicular to the string. The string is directed along the x axis. In the presence of a driving force, $V(u)$ can be represented as

$$V(u)=V_0(u)-Fu, \quad (2)$$

where $V_0(u)$ is a potential of a single pinning line of size u_0: $V_0(0)=0$ and $V_0(|u|\geqslant u_0)=E_0$. The driving force F renders the states localized near $u=0$ metastable and the string depins and leaves the defect in a finite time Γ^{-1}. The lifetime Γ^{-1} will be obtained analytically for the whole temperature range in the limit of small F.

At zero temperature the string has no energy to overcome the potential barrier that separates the states with $u=0$ and with $u\geqslant E_0/F$ where the barrier disappears. In this case the decay rate is determined by macroscopic quantum tunneling. To study the tunneling process one has to rewrite the action in imaginary time (for temperature T),

$$A[u(x,\tau)]=\int_{-\hbar/2T}^{\hbar/2T}d\tau\int dx\left[\frac{\rho}{2}\dot{u}^2+\frac{\rho c^2}{2}(\partial_x u)^2+V(u)\right], \quad (3)$$

and find the saddle point solution $\tilde{u}(x,\tau)$ for this action that can be obtained from variation of Eq. (3). Such a solution of a Euclidean field theory is commonly referred to as an instanton.[8] Then, to exponential accuracy the lifetime $\Gamma^{-1}\propto\exp(-A[\tilde{u}]/\hbar)$.

The action can be easily estimated as follows.[9,4] Let τ be the tunneling time. Then the mass involved in the tunneling process can be estimated as $M\sim\rho c\tau$. We can describe the collective tunneling process as a tunneling of one particle of mass M through a potential barrier of height $E_0 c\tau$. The tunneling time can be obtained from the condition that the kinetic energy $M(E_0/F)^2/\tau^2$ should be of the order of the potential energy $E_0 c\tau$. This gives the estimate $\tau^2 \sim \rho E_0/F^2$ and the action

$$A_Q \sim \frac{\rho c E_0^2}{F^2}. \tag{4}$$

Such a quasiclassical treatment is valid provided that $A_Q/\hbar \gg 1$. This criterion can be expressed as

$$\frac{\hbar}{\rho c u_0^2}\alpha^2 \ll 1, \tag{5}$$

where $\alpha = F/F_0$ and $F_0 \sim E_0/u_0$ is the typical value of the force that keeps the string in the well. F_0 is also of the order of the critical driving force F_c at which the potential barrier disappears. The following consideration will be constrained to the limit of small $\alpha \ll 1$.

At high temperatures when the periodicity $\hbar/T$ along the τ axis becomes smaller than the size $\sim(\rho E_0)^{1/2}/F$ of the quantum instanton, the saddle point solution $\tilde{u}(x)$ does not depend on τ any more and the escape rate is determined by pure thermal activation. In this case $\Gamma^{-1} \propto \exp(-\Delta/T)$, where Δ is the energy of the optimal configuration $\tilde{u}(x)$ that extremizes Eq. (3). Note that it is independent of the dynamics since the thermal saddle point solution does not depend on imaginary time. The energy barrier Δ can be estimated in a way similar to the one used for the quantum case. Dimensional analysis yields

$$\Delta \sim \frac{(\rho c^2 E_0^3)^{1/2}}{F}. \tag{6}$$

The activation energy diverges at zero driving force. Such a behavior should be contrasted with the thermal kink-antikink nucleation in the models with several degenerate minima (sine-Gordon, ϕ^4 theories),[1,2] where in the absence of a driving force the activation barrier remains finite and is twice as large as the kink rest energy E_K. In the present case, the single minimum of $V_0(u)$ is nondegenerate, resulting in large F-dependent size of the instanton and divergent Δ.

There exists a temperature T_c when the probabilities of quantum and thermal instanton nucleations become equal,

$$T_c \sim \hbar\frac{\Delta}{A_Q} \sim \frac{\hbar F}{(\rho E_0)^{1/2}}. \tag{7}$$

Usually, this temperature indicates a crossover from quantum tunneling to thermal activation. This is the case for single-particle tunneling,[7] tunneling of a string between two degenerate minima when a small driving force lifting the degeneracy is applied,[1,2] and quantum breaking of an elastic string.[4] In the present problem the situation is different. It will be shown that for $T < T_c$ the string escapes due to quantum tunneling, temperature corrections to the action being exponentially small for $\alpha \ll 1$. For $T > T_c$ the lifetime is determined by pure thermal activation. Hence, at $T = T_c$ the system exhibits a kind of first-order phase transition from the quantum to the thermal regime and no crossover region in the conventional sense is obtained. Such a behavior is a consequence of the coexistence of *two qualitatively different* quantum and thermal instantons in the vicinity of T_c. The total $\Gamma^{-1} = (\Gamma_Q + \Gamma_T)^{-1}$ is equal to Γ_Q^{-1} for $T < T_c$ and to Γ_T^{-1} for $T > T_c$. On the other hand, in a usual situation there exists only *one* saddle point solution of Eq. (3) that describes quantum tunneling at low temperatures, pure activation at high temperatures, and crosses over between them at $T \sim T_c$.

III. QUASICLASSICAL ANALYSIS

In this section the saddle point solutions for the action (3) are obtained in the limit of a small driving force. For further use I want to specify some quantities defined above by order of magnitude estimates. We will see below that in the limit of small F the quasiclassical results for the nucleation rate are independent of the certain features of $V_0(u)$ at the scale u_0. The only relevant parameter is the depth E_0 of the potential. The other characteristic of $V_0(u)$ that determines instantons interaction and the strength of quantum fluctuations is the curvature at the origin $V_0''(0)$. Instead of it I define the width of the pinning potential according to

$$u_0^2 = \frac{2E_0}{V_0''(0)}. \tag{8}$$

The ratio of the driving force to the pinning force is the small parameter in the problem:

$$\alpha = \frac{Fu_0}{E_0} = \left[\frac{2F^2}{E_0 V_0''(0)}\right]^{1/2}. \tag{9}$$

In the following I will work in the units $\rho = c = 1$. One has to rescale $x \to (\rho c^2)^{-1/2} x, \tau \to \rho^{-1/2}\tau$ to return to dimensional units. The classical equation of motion obtained from the action (3) is

$$(\partial_x^2 + \partial_\tau^2)u = V'(u). \tag{10}$$

The function $u(x,\tau)$ is periodic in τ with the period $\hbar/T$ and vanishes at $|x| \to \infty$. Equation (10) defines a two-dimensional (2D) nonlinear electrostatics problem, the charge density being dependent on the electrical potential u.

The solution of Eq. (10) behaves essentially different for $u \ll u_0$ and for $u \gg u_0$ [see Eq. (2) for the definition of $V(u)$]. Let us define a curve Γ on the (x,τ) plane where $u = u_0$. It separates the plane into inner and outer domains $\mathcal{B}$ and $\mathcal{B}$. In the outer domain $\mathcal{B}$, $u < u_0$, which corresponds to the string lying in the pinning well $V_0(u)$. Inside the domain $\mathcal{B}$, $u > u_0$ and the string is under the barrier.

The key point in the determination of $u(x,\tau)$ from the nonlinear Eq. (10) is to obtain the solution in the outer domain $\mathcal{B}$ and to extract the boundary condition for the internal region of the instanton. The size of the instanton [R for the quantum instanton (16) or L for the thermal instanton (18); see Fig. 1 for geometry clarifying] is of the order of $E_0^{1/2}/F$ as estimated above. In the external region $\mathcal{B}$ of the instanton, u rapidly falls off to zero at a distance a that can be estimated from Eq. (10) as $a \simeq [V''(0)]^{-1/2} \sim \alpha R$.

Since a is much smaller than the boundary curvature radius (R for the quantum instanton, or ∞ for the thermal instanton), one can neglect the curvature in the Laplace operator in a layer of width a along the boundary Γ of the instanton and write simply $\nabla^2 \simeq \partial^2/\partial n^2$, where $\mathbf{n}$ is the out-

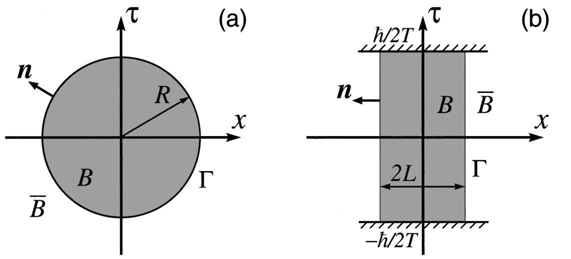

FIG. 1. The geometry of the quantum (a) and thermal (b) instantons.

ward normal to the boundary of the instanton. Then Eq. (10) becomes the equation of motion of a classical particle in an inverted potential $-V(u)$:

$$\frac{d^2u}{dn^2}=V'(u). \tag{11}$$

The energy $\mathcal{E}=1/2(du/dn)^2-V(u)$ is an integral of the motion. Its value should be obtained from the condition at infinity, which gives $\mathcal{E}=0$. Applied to $u=u_0$, the equation $\mathcal{E}=0$ gives the boundary condition for the interior problem:

$$\left.\frac{du}{dn}\right|_{\Gamma}=-\sqrt{2E_0}. \tag{12}$$

The other condition is $u|_{\Gamma}=u_0$. Thus, we arrive at the following problem:

$$\nabla^2u=-F, \tag{13}$$

$$u|_{\Gamma}=u_0, \tag{14}$$

$$\left.\frac{du}{dn}\right|_{\Gamma}=-\sqrt{2E_0}. \tag{15}$$

Equations (13) and (14) define a Dirichlet problem with a unique solution for a given contour Γ. The solution of Eq. (13) with the boundary condition (15) (von Neumann problem) is unique as well. The problems (13)–(15) are therefore strongly overdetermined and cannot be solved for a general contour Γ.

There are only two regions for which the solution of Eq. (13) satisfies both boundary conditions simultaneously. They are (i) the circle and (ii) the strip (see Fig. 1).

(i) Circle of radius R. The solution is

$$u=u_0+\frac{F}{4}(R^2-r^2), \tag{16}$$

where $r^2=x^2+\tau^2$. The radius can be obtained from Eq. (12):

$$R^2=\frac{8E_0}{F^2}. \tag{17}$$

This is the quantum instanton.

(ii) Strip of width $2L$ parallel to the τ axis. Then

$$u=u_0+\frac{F}{2}(L^2-x^2), \tag{18}$$

with

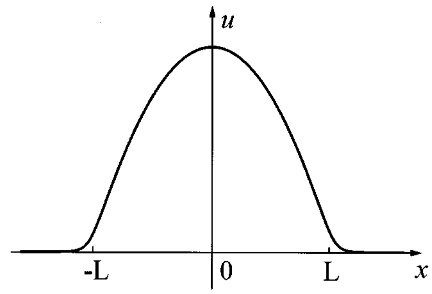

FIG. 2. The form of the thermal instanton. $u(x)$ is given by Eq. (18) for $|x|<L$ and exponentially decays for $|x|>L$. The figure can also be considered as the radial dependence $u(r)$ of the quantum instanton (16).

$$L^2=\frac{2E_0}{F^2}. \tag{19}$$

This solution is time independent and holds at large temperatures.

The solutions are shown schematically in Fig. 2.

To compute the action one has to know the solution of Eq. (10) everywhere. The contribution of the boundary layer to the action is not universal and depends on the features of the pinning potential $V_0(u)$ but is of order α when compared to the contribution from the interior region. Thus, to leading order in α, the action for the circle is (here and below dimensional units are restored)

$$A_Q=4\pi\frac{\rho c E_0^2}{F^2}, \tag{20}$$

whereas for the strip (temperature T) the result reads

$$A_T(T)=\hbar\frac{\Delta_0}{T}, \tag{21}$$

with Δ_0 the energy cost of the optimal tongue (18),

$$\Delta_0=\frac{2^{5/2}\rho^{1/2}cE_0^{3/2}}{3F}. \tag{22}$$

Quantum and thermal actions become equal at

$$T_c=\frac{2^{1/2}}{3\pi}\frac{\hbar F}{(\rho E_0)^{1/2}}. \tag{23}$$

The strip (18) is a solution for all T. It will be shown in the next section that this solution becomes unstable below some temperature $T_0=0.9T_c$. The circle solution exists as long as its diameter $2R$ is less than the periodicity $\hbar/T$ in τ. The quantum solution then ceases to exist at

$$T_1=\frac{3\pi}{8}T_c. \tag{24}$$

The dependence of the actions on temperature for the two solutions is illustrated in Fig. 3(a).

The lifetime Γ^{-1} is determined by the solution with the minimal action. To exponential accuracy,

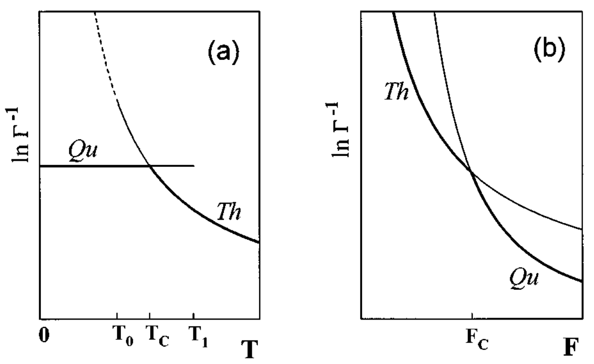

FIG. 3. (a) The dependence of the actions of the quantum (Qu) and thermal (Th) instantons on temperature at constant F. The decay rate exhibits a transition from quantum tunneling to thermal activation at $T=T_c$. (b) Decay rate vs F at constant T. $F=F_c$ separates the quantum and thermal regimes.

$$\ln\Gamma^{-1}\propto\begin{cases} A_Q/\hbar & \text{for } T<T_c,\\ \Delta_0/T & \text{for } T>T_c.\end{cases} \tag{25}$$

This answer holds for all temperatures but not too close to T_c. In a very narrow region around T_c, where $|1-T/T_c|<T_c/\Delta\sim\alpha^2(\hbar/\rho c u_0^2)\ll 1$ [cf. Eq. (5)], both solutions contribute to Γ^{-1} according to the formula $\Gamma^{-1}=(\Gamma_Q+\Gamma_T)^{-1}$. Note that the last equation is not a bridging formula but an exact expression up to exponentially small corrections. This is a consequence of the coexistence of two types of instantons close to T_c.

It is also instructive to study the temperature dependence of the quantum tunneling probability. It appears to be very small due to the exponential decay of u at large distances. Actually, the distance between the boundary of the instanton (16) and its image is $\delta\tau=\hbar/T-\hbar/T_1$. Then according to Eq. (10) the overlap of the exponentially decaying tails of the instanton and its image is as small as $u_0\exp[-(V''(0)/\rho)^{1/2}\delta\tau]$, resulting in

$$\frac{\delta A_Q(T)}{A_Q}\sim\exp\left[-\left(\frac{2E_0}{\rho u_0^2}\right)^{1/2}\left(\frac{\hbar}{T}-\frac{\hbar}{T_1}\right)\right]. \tag{26}$$

In the vicinity of T_1,

$$\frac{\delta A_Q(T)}{A_Q}\sim\exp\left[-\frac{8}{\alpha}\left(1-\frac{T}{T_1}\right)\right]. \tag{27}$$

IV. FLUCTUATIONS

Let us now take fluctuations near the classical instantons into account and determine the preexponential factors in Eq. (25). The starting point is the expression for the lifetime Γ^{-1},[10,3]

$$\hbar\Gamma=2T\frac{\mathrm{Im}Z}{\mathrm{Re}Z}, \tag{28}$$

where Z is the partition function,

$$Z=\int Du(x,\tau)\exp\left(-\frac{A[u]}{\hbar}\right). \tag{29}$$

The neighborhood of the saddle point solution $\tilde{u}(x,\tau)$ of the equation $\delta A[u]/\delta u=0$ gives the main contribution to the imaginary part of Z. I expand u near $\tilde{u}$ according to

$$u(x,\tau)=\tilde{u}(x,\tau)+\sum_\alpha C_\alpha u_\alpha(x,\tau), \tag{30}$$

where $u_\alpha(x,\tau)$ are the eigenfunctions of the operator $\delta^2A[u]/\delta u^2$:

$$(\delta^2A[u]/\delta u^2)_{\tilde{u}}u_\alpha(x,\tau)=\lambda_\alpha u_\alpha(x,\tau). \tag{31}$$

The lowest eigenvalue λ_0 is negative, resulting in the imaginary contribution to the partition function. $\lambda_1=0$ is also an eigenvalue of Eq. (31) related to the zero mode with respect to translation. For the thermal instanton (18), $\lambda_1=0$ is nondegenerate and $u_1(x)\propto\partial_x\tilde{u}$ corresponds to the zero mode along the x axis, while for the quantum instanton (16), the zero level is twofold degenerate since the location of the instanton on the x,τ plane depends on both coordinates.

Applying standard methods[11,12,3] for zero modes and the negative eigenvalues we arrive at the following expressions: The lifetime per unit length for thermal activation is

$$\frac{\Gamma}{L}=\left[\frac{T}{2\pi\hbar^2}\int dx\left(\frac{\partial\tilde{u}}{\partial x}\right)^2\right]^{1/2}\left|\frac{\det'(\delta^2A[u]/\delta u^2)_{u=\tilde{u}}}{\det(\delta^2A[u]/\delta u^2)_{u=0}}\right|^{-1/2}\times\exp\left(-\frac{A[\tilde{u}]}{\hbar}\right). \tag{32}$$

The decay rate for quantum tunneling reads

$$\frac{\Gamma}{L}=\frac{1}{2\pi\hbar}\left[\iint dx\,d\tau\left(\frac{\partial\tilde{u}}{\partial x}\right)^2\right]^{1/2}\left[\iint dx\,d\tau\left(\frac{\partial\tilde{u}}{\partial\tau}\right)^2\right]^{1/2}\times\left|\frac{\det''(\delta^2A[u]/\delta u^2)_{u=\tilde{u}}}{\det(\delta^2A[u]/\delta u^2)_{u=0}}\right|^{-1/2}\exp\left(-\frac{A[\tilde{u}]}{\hbar}\right). \tag{33}$$

Here $\det'$ and $\det''$ denote the determinants with the zero eigenvalues omitted.

A. High-temperature instanton

For the high-temperature τ-independent solution (18) (I use the units $\hbar=\rho=c=1$),

$$\frac{\delta^2A}{\delta u^2}=-\partial_\tau^2-\partial_x^2+U(x), \tag{34}$$

$$U(x)=\frac{\partial^2V(\tilde{u}(x))}{\partial u^2}. \tag{35}$$

The eigenvalues of the operator (34) are given by

$$\lambda_{mn}=\epsilon_m+(2\pi Tn)^2, \tag{36}$$

where n is an integer and $\{\epsilon_m\}$ is the spectrum of the one-dimensional Schrödinger equation

$$[-\partial_x^2+U(x)]\psi_m(x)=\epsilon_m\psi_m(x). \tag{37}$$

The potential $U(x)$ is illustrated schematically in Fig. 4. $L\propto\alpha^{-1}$ is the semiwidth of the instanton strip (18), and the width of the minimum of $U(x)$ near $|x|=L$, $a\sim L\alpha$, is independent of α; $U(|x|\gg L)=\omega^2\equiv V_0''(0)$ is the oscillator frequency for the pinning potential $V_0(u)$. The form of the potential near $|x|=L$ depends on the detailed features of $V_0(u)$. For $\alpha\to 0$, $L\to\infty$ and the potential well $U(x)$ becomes deep in the sense that as many as $(4/\pi)\alpha^{-1}$ discrete levels exist in this potential. Excited states can be obtained within the quasiclassical approximation. It is possible also to obtain the low-lying levels in the limit of small α.

Let us define γ as a function of the continuous parameter ϵ as

$$\gamma(\epsilon)=\partial_x \ln\psi(L), \tag{38}$$

where $\psi(x)$ is the solution of Eq. (37) with energy ϵ that vanishes at $x\to\infty$. Once the function $\gamma(\epsilon)$ is known, one can obtain the discrete spectrum in the following way. For $|x|<L$, the solutions of Eq. (37) are given by $\psi_\epsilon(x)=\cos(\sqrt{\epsilon}x)$ and $\psi_\epsilon(x)=\sin(\sqrt{\epsilon}x)$ for even and odd states, respectively. The spectrum then can be obtained from the continuity of the logarithmic derivative of $\psi_\epsilon(x)$ at $x=L$:

$$\partial_x \ln\psi_\epsilon(L)=\gamma(\epsilon). \tag{39}$$

The function $\gamma(\epsilon)$ is not universal and depends on the shape of the minimum of $U(x)$, that is, on the details of $V_0(u)$. It appears, however, that $\gamma(0)$ is independent of the pinning potential. We may find $\gamma(0)$ by using our knowledge that the first excited level has zero energy. For $|x|<L$ its wave function linearly depends on x, yielding

$$\gamma(0)=1/L. \tag{40}$$

The low-lying states can be obtained from Eq. (39) with the right-hand side set equal to $\gamma(0)$, thus neglecting the difference between $\gamma(\epsilon)$ and $\gamma(0)$ for small ϵ. Then one immediately obtains the implicit equations for the spectrum,

$$\sqrt{\epsilon}L\ \tan(\sqrt{\epsilon}L)=-1 \tag{41}$$

for even states and

$$\sqrt{\epsilon}L\ \cot(\sqrt{\epsilon}L)=1 \tag{42}$$

for odd states.

The approximation $\gamma(\epsilon)\simeq\gamma(0)$ is valid for small enough energies $\epsilon\ll\epsilon_*\sim\gamma(0)/\gamma'(0)$. One can estimate $\gamma'(0)$ as $\gamma(\omega^2)/\omega^2\sim 1/\omega$, yielding $\epsilon_*\sim\alpha^{-1}/L^2$. On the other hand, the energy of the nth excited level can be estimated as $\epsilon_n\simeq[\pi n/(2L)]^2$ for large n. Therefore, Eqs. (41) and (42) correctly describe as many as $\alpha^{-1/2}$ low-lying levels.

The negative eigenvalue ϵ_0 can be obtained from Eq. (41) and is equal to

$$\epsilon_0=-\frac{\mu^2}{L^2}, \tag{43}$$

where $\mu=1.199\,68$ is the solution of the equation $\mu\ \tanh\mu=1$.

The negative eigenvalue determines the temperature T_0 where λ_{01} changes the sign, leading to the instability of the thermal instanton (18) with respect to a small perturbation of its boundary. Inserting Eq. (43) into Eq. (36), I obtain

$$T_0=\frac{\mu}{2\pi}\frac{1}{L}, \tag{44}$$

in dimensional units,

$$T_0=\frac{\mu}{2^{3/2}\pi}\frac{\hbar F}{(\rho E_0)^{1/2}}=0.900T_c. \tag{45}$$

Now we are in a position to calculate the ratio of determinants in Eq. (32). To do it, one has to compute the products over n,[2] resulting in

$$\left|\frac{\det'(\delta^2A[u]/\delta u^2)_{u=\tilde{u}}}{\det(\delta^2A[u]/\delta u^2)_{u=0}}\right|^{-1/2}=\frac{T}{2\ \sin(\pi T_0/T)}e^{-\Delta_1/T}\chi\left(\frac{T}{T_0}\right), \tag{46}$$

with the quantum correction to the barrier height,

$$\Delta_1=\frac{1}{2}\sum_{m=2}^{M}\epsilon_m^{1/2}+\int_{-\infty}^{\infty}\frac{dq}{4\pi}\frac{\partial\delta(q)}{\partial q}\epsilon(q)^{1/2}. \tag{47}$$

Here $\delta(q)$ is the phase shift of the states in the continuous spectrum, $\epsilon(q)=\omega^2+q^2$, and M labels the last discrete state. The function χ is defined according to

$$\ln\chi\left(\frac{T}{T_0}\right)=-\sum_{m=2}^{M}\ln(1-e^{-\epsilon_m^{1/2}/T})-\int_{-\infty}^{\infty}\frac{dq}{2\pi}\frac{\partial\delta(q)}{\partial q}\times\ln(1-e^{-\epsilon(q)^{1/2}/T}). \tag{48}$$

At large q, $\delta(q)$ can be calculated perturbatively,[13]

$$\delta(q)=-\frac{1}{2q}\int[U(x)-\omega^2]dx=\frac{\omega^2L}{q}, \tag{49}$$

leading to a logarithmic divergence of the integral over continuous spectrum in Eq. (47) that should be cut off at some wave vector Λ:

$$\Delta_1=-\frac{\omega^2L}{4\pi}\ln\frac{\Lambda}{\omega}. \tag{50}$$

The ultraviolet cutoff Λ is a new parameter that should be introduced for the problem described by the Lagrangian (1) to be well defined. The value of Λ remains undefined in the model in question. Physically, it is determined by the length scale at which the elastic approximation of the bending energy in Eq. (1) breaks down. Then Λ may be estimated as the inverse core radius of the linear manifold considered.

$\chi(T/T_c)$ is a complicated function of temperature. For T close to T_0, only the lowest level with positive energy ϵ_2 is excited. Its energy can be obtained from Eq. (41): $\epsilon_2=(2.798\,39/L)^2$, resulting in

$$\chi(x\sim 1)\simeq 1+\exp\left(-\frac{14.7}{x}\right). \tag{51}$$

For intermediate temperatures $T_0\ll T\ll T_0/\alpha\sim\omega$, many levels contribute to the sum in Eq. (48). In this limit, $\epsilon_m\sim[\pi m/(2L)]^2$ and

$$\chi(x)=\exp\left(\frac{\mu}{6}x\right). \tag{52}$$

For $T \gg T_0$, many levels are excited and the quasiclassical approach can be applied. The sum over the states in Eq. (48) can be expressed via an integral over phase space:

$$\ln\chi\left(\frac{T}{T_0}\right) \simeq -2\ \ln(1-e^{-T_0/T}) + \frac{1}{2\pi}\int dx\ dp \times[\ln(1-e^{-\sqrt{\omega^2+p^2}/T}) - \ln(1-e^{-\sqrt{U(x)+p^2}/T})] = -2\ \ln(1-e^{-T_0/T}) + \frac{2L}{\pi}\int_0^\infty dp \times[\ln(1-e^{-\sqrt{\omega^2+p^2}/T}) - \ln(1-e^{-p/T})]. \quad (53)$$

The first term in Eq. (53) accounts for the two omitted states in Eq. (48). For $T \ll T_0/\alpha$, Eq. (53) reduces to Eq. (52). For $T \gg T_0/\alpha$, one finds

$$\chi(x) = \frac{1}{x^2}\exp\left(\frac{2}{\alpha}\right). \quad (54)$$

Substituting Eqs. (18) and (46) into Eq. (32), we obtain

$$\frac{\Gamma}{L} = B(T)\exp\left(-\frac{\Delta}{T}\right). \quad (55)$$

$\Delta = \Delta_0 + \Delta_1$ and $B(T)$ in conventional units are given by the following expressions:

$$\Delta = \frac{2^{5/2}\rho^{1/2}cE_0^{3/2}}{3F}\left\{1 - \frac{3\hbar}{8\pi\rho c u_0^2}\ln\left[\Lambda u_0\left(\frac{\rho c^2}{E_0}\right)^{1/2}\right]\right\}, \quad (56)$$

$$B(T) = \frac{1}{2^{1/4}3^{1/2}\pi^{1/2}}\frac{\rho^{1/4}E_0^{3/4}}{\hbar^2 c^{1/2}F^{1/2}}\frac{T^{3/2}}{\sin(\pi T_0/T)}\chi\left(\frac{T}{T_0}\right). \quad (57)$$

B. Quantum instanton

Fluctuations of the quantum instanton (16) are governed by the Hamiltonian

$$\frac{\delta^2 A}{\delta u^2} = -\partial_\tau^2 - \partial_x^2 + U(r), \quad (58)$$

where $U(r)$ is cylindrically symmetrical,

$$U(r) = \frac{\partial^2 V(\tilde{u}(r))}{\partial u^2}. \quad (59)$$

The potential $U(r)$ looks very similar to the one-dimensional potential $U(x \geq 0)$ illustrated in Fig. 4.

The ratio of determinants in Eq. (33) can be expressed in terms of the spectrum as

$$\left|\frac{\det''(\delta^2 A[u]/\delta u^2)_{u=\tilde{u}}}{\det(\delta^2 A[u]/\delta u^2)_{u=0}}\right|^{-1/2} = \exp\left\{-\frac{1}{2}\sum_m{}'' \ \ln|\epsilon_m| + \frac{1}{2}\sum_n \ \ln|\epsilon_n|\right\}, \quad (60)$$

where the first sum is taken over the states in the potential (59) and the second in the potential $U(r) = \omega^2$.

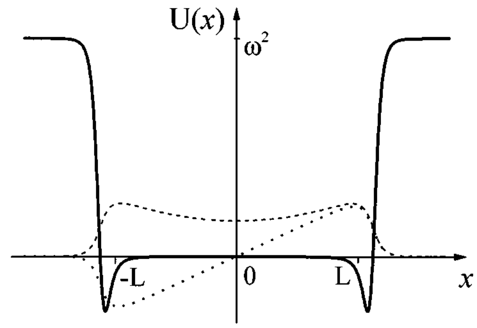

FIG. 4. Potential energy for the problem (37) linearized in the vicinity of the thermal instanton. Also shown are the wave functions of the ground (dashed line) and the first excited (dotted line) states.

We can estimate Eq. (60) within the quasiclassical approach by taking advantage of the large number of bound states in the potential (59) for small α. According to the quasiclassical quantization rule, each state occupies the volume $(2\pi)^2$ in phase space and Eq. (60) reduces to

$$\epsilon\ \exp\left\{-\frac{1}{2}\frac{1}{(2\pi)^2}\int d^2r\ d^2p[\ln\sqrt{U(r)+p^2} - \ln\sqrt{\omega^2+p^2}]\right\} = \epsilon\ \exp\left\{\frac{R^2}{8}\int_0^\infty pdp\ \ln\frac{\omega^2+p^2}{p^2}\right\}. \quad (61)$$

The integral over the momentum is logarithmically divergent, leading to

$$\epsilon\ \exp\left(\frac{R^2\omega^2}{8}\ln\frac{\Lambda}{\omega}\right). \quad (62)$$

Inserting Eq. (62) into Eq. (33), and using Eq. (16) rewritten in conventional units, I obtain the final result

$$\frac{\Gamma}{L} = \frac{E_0}{\hbar}\exp\left(-\frac{A}{\hbar}\right), \quad (63)$$

$$A = 4\pi\frac{\rho c E_0^2}{F^2}\left\{1 - \frac{\hbar}{2\pi\rho c u_0^2}\ln\left[\Lambda u_0\left(\frac{\rho c^2}{E_0}\right)^{1/2}\right]\right\}. \quad (64)$$

The results (56) and (64) are valid provided that the fluctuation contribution is less than the classical one. This provides the condition

$$\frac{\hbar}{2\pi\rho c u_0^2}\ln\left[\Lambda u_0\left(\frac{\rho c^2}{E_0}\right)^{1/2}\right] \ll 1. \quad (65)$$

This is a much stronger condition than that of Eq. (5). When Eq. (65) fails, then one has to take unharmonic fluctuations into account that will renormalize the numerical coefficient in front of Eq. (64), whereas the estimate (4) holds as long as the condition (5) is fulfilled.

V. CONCLUSION

The problem of a massive string depinning from a linear defect has been solved in the limit of small driving force ($\alpha \ll 1$) allowing for an exact analytical solution in the whole temperature range. It has been shown that there exists a temperature region $T_0 < T < T_1$ where the action (3) has *two* different saddle point solutions: Eqs. (16) and (18). The actions for these solutions become equal at $T = T_c$. For $T < T_c$, the lifetime is determined by quantum tunneling [Eq. (63)] with exponentially small temperature corrections (27). For $T > T_c$, the lifetime is given by a pure activation expression (55). At $T = T_c$ the system jumps from quantum to classical behavior. The dependence of $\ln\Gamma^{-1}$ vs T at constant F is shown in Fig. 3(a). The possibility of such a first-order transition between quantum and thermal behavior was first discovered in Ref. 14.

It is worth emphasizing that the first order transition is obtained for small $\alpha \ll 1$. As α increases, $T_0(\alpha)$ and $T_1(\alpha)$ come closer to each other and become equal at some $\alpha_c \sim 1$; that is, there exists a tricritical point where the first-order transition disappears. For α close but below α_c, the temperature corrections to the quantum tunneling rate become large and the transition at $T_c(\alpha)$ should be considered as a transition from thermally assisted quantum tunneling to pure activation.

Besides the transition at constant F, one can consider the transition at constant T by changing the applied force which may be easier to achieve experimentally. $\ln\Gamma^{-1}$ as a function of F is shown in Fig. 3(b). For

$$F < F_c = \frac{3\pi}{2^{1/2}} \frac{T}{\hbar} (\rho E_0)^{1/2},$$

the metastable state decays via thermal activation with $\ln\Gamma^{-1} \propto F^{-1}$, whereas for $F > F_c$, the string escapes via quantum tunneling with $\ln\Gamma^{-1} \propto F^{-2}$.

The problem in question is connected with the nucleation phenomenon in two-dimensional first-order quantum phase transitions. Consider a first-order phase transition controlled by some parameter x. The quantum nature of the transition implies that it exists even at $T = 0$. One possible example is the liquid-solid transition in 2D helium films. Let $x > 0$ correspond to phase I and $x < 0$ to phase II. The phase boundary is a string that has mass density ρ and elasticity $\rho c^2 = \sigma$, where σ is the boundary line tension. The difference in chemical potentials of the phases is equivalent to the driving force $F = \mu_{\mathrm{I}} - \mu_{\mathrm{II}} \propto x$. Suppose that phase II is prepared at $x > 0$. Such a state is metastable and is destroyed by the nucleation of phase I that can take place near the edges of the 2D system or in the bulk. Nucleation near the edges is exactly the problem of a massive string depinning from a linear defect. In the present case, the edge of the system plays the role of the pinning well with the depth $E_0 = \sigma$. It can be shown that nucleation always occurs near the edges since the bulk tunneling action and activation barrier are larger than the corresponding values for the edge: $A_{\mathrm{bulk}}/A_{\mathrm{edge}} = 3\pi/2^{5/2}$ and $\Delta_{\mathrm{bulk}}/\Delta_{\mathrm{edge}} = 16\sqrt{2}/15$.

Finally, the results (56) and (64) for the string on a plane can be easily generalized to the string in $(1+d)$ dimensions. The instantons (16) and (18) remain the same since any transverse displacement will cost extra bending energy. The preexponentials will be different; they will be multiplied by an additional factor from transverse fluctuations. The main effect is the appearance of a coefficient d in front of the logarithm in formulas (56) and (64).

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