PHYSICAL REVIEW B **87**, 020501(R) (2013)

Onset of superconductivity in a voltage-biased normal-superconducting-normal microbridge

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(Received 10 September 2012; revised manuscript received 19 December 2012; published 2 January 2013)

We study the stability of the normal state in a mesoscopic NSN junction biased by a constant voltage V with respect to the formation of the superconducting order. Using the linearized time-dependent Ginzburg-Landau equation, we obtain the temperature dependence of the instability line, $V_{\text{inst}}(T)$, where nucleation of superconductivity takes place. For sufficiently low biases, a stationary symmetric superconducting state emerges below the instability line. For higher biases, the normal phase is destroyed by the formation of a nonstationary bimodal state with two superconducting nuclei localized near the opposite terminals. The low-temperature and large-voltage behavior of the instability line is highly sensitive to the details of the inelastic relaxation mechanism in the wire. Therefore, experimental studies of $V_{\text{inst}}(T)$ in NSN junctions may be used as an effective tool to access the parameters of the inelastic relaxation in the normal state.

DOI: 10.1103/PhysRevB.87.020501 PACS number(s): 74.40.Gh, 72.10.Di, 72.15.Lh, 74.78.Na

I. INTRODUCTION

Nonequilibrium superconductivity has being attracting significant experimental and theoretical attention over the past few decades,[1–3] ranging from vortex dynamics[4] to the physics of the resistive state in current-carrying superconductors.[5–9] It was recognized long ago[10] that a superconducting wire typically has a hysteretic current voltage characteristic specified by several "critical" currents. In an up-sweep, a current exceeding the thermodynamic depairing current, $I_c(T)$, does not completely destroy superconductivity but drives the wire into a nonstationary resistive state,[11] with the excess phase winding relaxing through the formation of phase slips.[12] The resistive state continues until $I_2(T) > I_c(T)$, when the wire eventually becomes normal. In the down-sweep of the current voltage characteristic, the wire remains normal until $I_1(T) < I_2(T)$ when an emerging order parameter leads to the reduction of the wire resistance.

The theoretical description of a nonequilibrium superconducting state is a sophisticated problem, requiring a simultaneous account of the nonlinear order parameter dynamics and quasiparticle relaxation under nonstationary conditions. The resulting set of equations is extremely complicated[1,4] and can be treated only numerically[13–15] (even then the stationarity of the superconducting state is often assumed for one-dimensional problems[13,14]). A more intuitive but somewhat oversimplified approach is based on the time-dependent Ginzburg-Landau (TDGL) equation for the order parameter field $\Delta(\mathbf{r},t)$. The TDGL approach can be justified only in a very narrow vicinity of the critical temperature, T_c, provided that the electron-phonon (e-ph) interaction is sufficiently strong.[16] These generalized TDGL equations are analyzed numerically in Refs. 5 and 17.

While the applicability of the TDGL equation in the superconducting region is a controversial issue, its linearized form can be safely employed to find the line $I_{\text{inst}}(T)$ of the absolute instability of the normal state with respect to the appearance of an infinitesimally small order parameter $\Delta(\mathbf{r},t)$.[10,18,19] If the transition to the superconducting state is second order, then $I_1(T)$ coincides with $I_{\text{inst}}(T)$. Otherwise the actual instability takes place at a larger $I_1(T) > I_{\text{inst}}(T)$. In both cases, $I_{\text{inst}}(T)$ gives the lower bound for $I_1(T)$.

Previous results[10,18] for the instability line of a superconducting wire connected to normal reservoirs (NSN microbridge) have been obtained in the limit of quasiequilibrium. This approximation breaks down for low-T_c superconducting wires shorter than the e-ph relaxation length, $l_{e\text{-ph}}(T_c)$ [e.g., for aluminum, $l_{e\text{-ph}}(T_c) \approx 40\ \mu$m (Ref. 20)]. Such systems have recently been experimentally studied in Refs. 14 (Al), 21, and 22 (Zn; reservoirs may be driven normal by a magnetic field). It was found that for sufficiently large biases, superconductivity arises near the terminals through a second-order phase transition, with $I_1(T) = I_{\text{inst}}(T)$.[14]

In this paper, we study the normal state instability line in an NSN microbridge biased by a dc voltage V, relaxing the assumption of strong thermalization. For small biases, $eV \ll T_c$, the instability line is universal and we reproduce the results of Refs. 10 and 18. The universality breaks down for larger biases, where we obtain $V_{\text{inst}}(T)$ *as a functional of the normal state distribution function* and analyze it for various types of inelastic interactions.

We model the NSN microbridge as a diffusive wire of length L coupled at $x = \pm L/2$ to large normal reservoirs via transparent interfaces. The terminals are biased by a constant voltage V. The wire length, L, is assumed to be larger than the zero-temperature coherence length, $\xi_0 = \sqrt{\pi D/8T_{c0}}$, where D is diffusion coefficient and T_{c0} is the critical temperature of the infinite wire. The equilibrium critical temperature, $T_c = T_{c0}(1 - \pi^2\xi_0^2/L^2)$, is smaller than T_{c0} due to the finite-size effect.[23]

II. GENERAL STABILITY CRITERION

An arbitrary nonequilibrium normal state becomes absolutely unstable with respect to superconducting fluctuations if an infinitesimally small order parameter, $\Delta(\mathbf{r},t)$, does not decay with time. For stability analysis it suffices to describe the evolution of $\Delta(\mathbf{r},t)$ by the linearized TDGL equation. For a dirty superconductor, the latter can be readily derived from the Keldysh σ-model formalism[24–26] or dynamic Usadel equations[4] by expanding in Δ. It takes the form $(L_R)^{-1} * \Delta = 0$, where $(L^R)^{-1}$ is the inverse fluctuation propagator, and convolution in time and space indices is implied. In the frequency representation, $(L^R_\omega)^{-1}$ is an integral operator with the kernel

$$\left(L^R_\omega\right)^{-1}_{\mathbf{r},\mathbf{r}'} = -\frac{\delta_{\mathbf{r},\mathbf{r}'}}{\lambda} + i\int_{-\omega_D}^{\omega_D} dE\, F(E,\mathbf{r})\, C_{\omega-2E}(\mathbf{r},\mathbf{r}'), \quad (1)$$

where λ is the dimensionless BCS interaction constant, ω_D is the Debye frequency, and C stands for the retarded Cooperon, $C_\varepsilon(\mathbf{r},\mathbf{r}') = \langle \mathbf{r}|(-D\nabla^2 - i\varepsilon)^{-1}|\mathbf{r}'\rangle$, vanishing at the boundary with the terminals.

The operator (1) depends on the normal-state nonequilibrium electron distribution function, $F(E,\mathbf{r})$. The latter should be determined from the kinetic equation

$$D\nabla^2 F(E,\mathbf{r}) + \mathcal{I}^{e\text{-}e}[F] + \mathcal{I}^{e\text{-ph}}[F] = 0, \quad (2)$$

with $\mathcal{I}^{e\text{-}e}[F]$ and $\mathcal{I}^{e\text{-ph}}[F]$ being the electron-electron (e-e) and e-ph collision integrals, respectively. The corresponding energy relaxation lengths, $l_{e\text{-}e}(T) \propto T^{-1/4}$ and $l_{e\text{-ph}}(T) \propto T^{-3/2}$, behave as a negative power of the temperature T in quasiequilibrium.[20] In the absence of inelastic collisions, the kinetic equation (2) is solved by the "two-step" function:[27,28]

$$F(E,x) = (1/2 - x/L)F_L(E) + (1/2 + x/L)F_R(E). \quad (3)$$

The distribution functions in the terminals, $F_{L,R}(E) = F_0(E \pm eV/2)$, are given by the equilibrium distribution function, $F_0(E) = \tanh(E/2T)$, shifted by $\pm eV/2$ ($e > 0$). In the opposite case of strong inelastic relaxation, the distribution function takes the form

$$F_{\text{in}}(E,x) = \tanh\{[E - e\phi(x)]/2T(x)\}, \quad (4)$$

where $\phi(x) = Vx/L$ is the potential in the normal state and $T(x)$ is the effective temperature. For strong lattice thermalization ($l_{e\text{-ph}} \ll L \ll l_{e\text{-}e}$), $T(x) = T$. For the dominating e-e scattering ($l_{e\text{-}e} \ll L \ll l_{e\text{-ph}}$), $T^2(x) = T^2 + (3/4\pi^2)[1 - (2x/L)^2](eV)^2$.[27]

The evolution governed by the operator (1) can be naturally described in terms of the eigenmodes $\Delta_k(\mathbf{r})e^{-i\omega_k t}$ annihilated by $(L^R_\omega)^{-1}$. The normal state is stable provided $\operatorname{Im}\omega_k < 0$ for all eigenmodes. Generally, the spectrum can be obtained only numerically. Analytical treatment is possible if Eq. (1) may be linearized in ω: $(L^R_\omega)^{-1} = i\tau\omega - \mathcal{H}$. The instability occurs when the real part of the lowest eigenvalue of $\mathcal{H}$ turns to zero.

The linearized fluctuation propagator (1) determines the instability line in the mean-field approximation. Beyond that, it is responsible for superconducting fluctuations which are neglected below assuming that the corresponding Ginzburg number is small.[29]

III. WEAK-NONEQUILIBRIUM REGIME

In the limit of low biases, $eV \ll T_c$, the deviation from equilibrium is small everywhere in the wire and the distribution function acquires a universal form, $F(E,x) \approx F_0(E) - F_0'(E)\, e\phi(x)$, regardless of the relaxation mechanism. Then Eq. (1) takes the form $(L^R_\omega)^{-1} = i\pi\omega/8T - \ln(T/T_{c0}) - (\xi_0/L)^2\, \mathcal{H}_v$, with

$$\mathcal{H}_v = -\partial_{\tilde{x}}^2 + 2iv\tilde{x}, \quad \tilde{x} \in [-1/2, 1/2]. \quad (5)$$

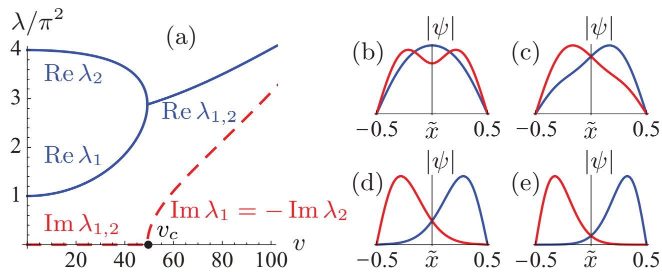

FIG. 1. (Color online) (a) Real (solid blue line) and imaginary (dashed red line) parts of the lowest eigenvalues $\lambda_{1,2}(v)$ of the Hamiltonian (5). The spectrum is entirely real until $v = v_c \approx 49.25$. (b)–(e) Spatial dependence of the absolute values of the eigenfunctions, $|\psi_1(\tilde{x})|$ (blue) and $|\psi_2(\tilde{x})|$ (red), for $v = 0.8v_c$, $1.2v_c$, $5v_c$, and $10v_c$, respectively.

The Hamiltonian $\mathcal{H}_v$ describes quantum-mechanical motion *in an imaginary electric field*, $v = eV/E_{\text{Th}}$ ($E_{\text{Th}} = D/L^2$ is the Thouless energy), on the interval $\tilde{x} \equiv x/L \in [-1/2, 1/2]$ with hard-wall boundary conditions, $\psi(\pm 1/2) = 0$. The Hamiltonian (5) has been recently analyzed in Ref. 18. It belongs to a class of non-Hermitian Hamiltonians invariant under the combined action of the time-reversal, $\mathcal{T}$: $f(x) \mapsto f^*(x)$, and parity, $\mathcal{P}$: $f(x) \mapsto f(-x)$, transformations. The $\mathcal{PT}$ symmetry of $\mathcal{H}_v$ ensures that its eigenvalues $\lambda_n(v)$ are either real or form complex-conjugated pairs.[30,31] At $v = 0$, the spectrum is nondegenerate: $\lambda_n(0) = \pi^2 n^2$ ($n = 1, 2, \ldots$). It evolves continuously with v, and a nonzero $\operatorname{Im}\lambda(v)$ arises only when the two lowest eigenvalues, $\lambda_1(v)$ and $\lambda_2(v)$, coalesce [see Fig. 1(a)]. This happens at $v = v_c \approx 49.25$,[18] indicating the transition to a complex-valued spectrum. For $v < v_c$, the ground state of (5) is $\mathcal{PT}$-symmetric, and hence $|\psi_1(\tilde{x})| = |\psi_1(-\tilde{x})|$. For $v > v_c$, the $\mathcal{PT}$ symmetry is spontaneously broken and there is a pair of states with the lowest $\operatorname{Re}\lambda(v)$: $\psi_L(\tilde{x}) = \psi_1(\tilde{x})$ and $\psi_R(\tilde{x}) = \psi_2(\tilde{x}) = \psi_1^*(-\tilde{x})$, shifted to the left (right) from the midpoint [see Figs. 1(b)–1(e)].

Spontaneous breaking of the $\mathcal{PT}$ symmetry associated with the spectral bifurcation at $v = v_c$ explains the appearance of asymmetric superconducting states observed in numerical simulations[32] and recent experiments.[14] The normal-state instability line, $V_{\text{inst}}(T)$, is specified implicitly by the relation

$$1 - T/T_{c0} = (\xi_0/L)^2 \operatorname{Re}\lambda_1[eV_{\text{inst}}(T)/E_{\text{Th}}], \quad (6)$$

and exhibits a singular behavior at the critical bias $eV_* = v_c E_{\text{Th}} \approx 50 E_{\text{Th}}$ (see the inset in Fig. 2). The bifurcation of the instability line occurs at the temperature $T_* \approx T_{c0}(1 - 28.44\,\xi_0^2/L^2)$. For long wires ($L \gg \xi_0$), T_* is very close to T_c.

The time dependence of the emergent superconducting state is determined by $\operatorname{Im}\lambda_1(v)$. Below the bifurcation threshold, for $V_{\text{inst}}(T) < V_*$, the system undergoes at $V = V_{\text{inst}}(T)$ the transition to a *stationary* superconducting state, with the superconducting chemical potential being the half-sum of the chemical potentials in the terminals. This state is supercurrent-carrying, and can withstand a maximum phase winding of π achieved at the critical bias V_*. For larger voltages, $V_{\text{inst}}(T) > V_*$, two modes, $\psi_L(x)$ and $\psi_R(x)$, nucleate simultaneously at $V_{\text{inst}}(T)$. The resulting bimodal superconducting state is *nonstationary*, and the left and

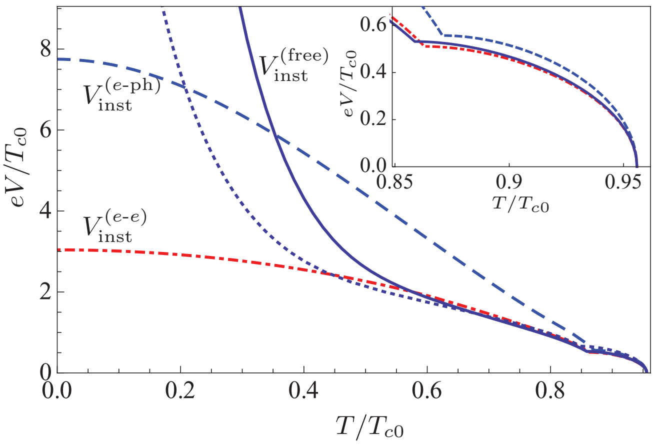

FIG. 2. (Color online) Instability voltage as a function of temperature, $V_{\rm inst}(T)$, obtained numerically for a wire of length $L = 15\xi_0$ for three limiting types of the distribution function: without inelastic relaxation (solid blue line) and with dominant e-e (dot-dashed line) or e-ph (dashed line) relaxation. The dotted curve illustrates the suppression of $V_{\rm inst}^{(\rm free)}(T)$ by a finite terminal resistance, $\beta = 0.1$ (see text). The inset shows the behavior in the vicinity of the bifurcation point.

right modes rotate with opposite frequencies, $\Omega_{L,R}(V) = \mp E_{\rm Th}\,{\rm Im}\,\lambda_1(eV/T)$, leading to an oscillating supercurrent in the wire.

IV. INCOHERENT REGIME

As the voltage is increased far above the bifurcation threshold, $V_{\rm inst}(T) \gg V_*$, the eigenmodes $\psi_{L,R}(x)$ gradually localize near the corresponding terminals, with their size, $a(V)$, becoming much smaller than the wire length [see Figs. 1(b)–1(e)]. This is the *incoherent* regime, where the overlap between $\psi_L(x)$ and $\psi_R(x)$ is exponentially small and nucleation of superconductivity near each terminal can be described independently.[14]

Using $a(V)/L$ as a small parameter and still working in the vicinity of T_c, we linearize $F(E,x)$ near the left terminal and reduce Eq. (1) to the form $(L_\omega^R)^{-1} = i\pi(\omega + eV)/8T - \ln(T/T_{c0}) - \mathcal{H}_\alpha$, where the operator

$$\mathcal{H}_\alpha = -\xi_0^2\partial_{x_L}^2 + \alpha x_L, \quad x_L \geqslant 0, \tag{7}$$

acts on the semiaxis $x_L \equiv x + L/2 \geqslant 0$ with the boundary condition $\psi(0) = 0$. The complex parameter α is a functional of the distribution function:

$$\alpha\left(\frac{eV}{T}\right) = -\int \frac{dE\,\partial_x F(E - eV/2,x)|_{x=-L/2+a(V)}}{2(E - i0)}. \tag{8}$$

Solving for the ground state of the Hamiltonian (7), we estimate the nucleus size as $a = (\xi_0^2/\alpha)^{1/3}$ (Ref. 10) and get for the instability line

$$1 - T/T_{c0} = \gamma_0\xi_0^{2/3}{\rm Re}\,\alpha^{2/3}[eV_{\rm inst}(T)/T], \tag{9}$$

where $-\gamma_0 \approx -2.34$ is the first zero of the Airy function. The left and right unstable states rotate with the frequencies $\Omega_{L,R}(V) = \mp[eV - \Omega_1(V)]$, where $\Omega_1(V) = (8T_{c0}/\pi)\gamma_0\xi_0^{2/3}\,{\rm Im}\,\alpha^{2/3}(eV/T)$ is a small correction to the Josephson frequency determined by the electrochemical potential of the corresponding terminal. At the instability line, the size a of the unstable mode is of the order of the temperature-dependent superconducting coherence length $\xi(T) \sim (1 - T/T_{c0})^{-1/2}\xi_0$.

For long wires ($L \gg \xi_0$), the incoherent regime partly overlaps with the weak-nonequilibrium regime. Then for $eV_* \ll eV_{\rm inst}(T) \ll T_c$, Eq. (9) gives a universal answer,

$$\frac{eV_{\rm inst}(T)}{T_{c0}} = \frac{2^{7/2}}{\pi}\frac{L}{\xi_0}\left(\frac{T_{c0} - T}{\gamma_0 T_{c0}}\right)^{3/2}, \tag{10}$$

which could have also been deduced from Eq. (6) at $v \gg 1$. Equation (9) exactly coincides with the result of Ref. 10 that superconductivity nucleates near the terminals at a finite current $I_{\rm inst}(T) \approx 0.356\,I_c(T)$.

The position of the instability line in the incoherent regime at large biases, $eV_{\rm inst}(T) \gg T_c$, depends on the relation between the inelastic lengths $l_{e\text{-}e}$ and $l_{e\text{-ph}}$, the wire length L, *and* the nucleus size $a(V)$. The presence of the latter scale, which probes the distribution function near the boundaries of the wire, leads to a rich variety of regimes realized at different temperatures.

For the three limiting distributions [Eqs. (3) and (4)], the function $\alpha(u)$ can be found analytically: (i) $\alpha_{\rm free}(u) = [\psi(1/2 + iu/2\pi) - \psi(1/2)]/L$ for the noninteracting case, $L \ll l_{e\text{-}e}, l_{e\text{-ph}}$, where $\psi(x)$ is the digamma function; (ii) $\alpha_{e\text{-ph}}(u) = i\pi u/4L$ for strong lattice thermalization, $l_{e\text{-ph}} \ll a(V) \ll L \ll l_{e\text{-}e}$; and (iii) $\alpha_{e\text{-}e}(u) = [i\pi u/4 + 3u^2/2\pi^2]/L$ for the dominant e-e interaction, $l_{e\text{-}e} \ll a(V) \ll L \ll l_{e\text{-ph}}$. In case (ii), the instability line $V_{\rm inst}^{e\text{-ph}}(T)$ is given by Eq. (10). In the vicinity of T_c, the instability lines in cases (i) and (iii) are given by

$$\frac{eV_{\rm inst}^{(\rm free)}(T)}{T_{c0}} = 1.13\exp\left\{\frac{L}{\xi_0}\left(\frac{T_{c0} - T}{\gamma_0 T_{c0}}\right)^{3/2}\right\}, \tag{11}$$

$$\frac{eV_{\rm inst}^{(e\text{-}e)}(T)}{T_{c0}} = \left(\frac{2\pi^2}{3}\frac{L}{\xi_0}\right)^{1/2}\left(\frac{T_{c0} - T}{\gamma_0 T_{c0}}\right)^{3/4}. \tag{12}$$

Counterintuitively, in cases (i) and (iii), the instability current $I_{\rm inst}(T) \propto V_{\rm inst}(T)/L$ has a nontrivial dependence on the system size, as opposed to Eq. (10). Such a behavior is a consequence of strong nonequilibrium in the wire. The limiting curves $V_{\rm inst}^{(\rm free)}(T)$, $V_{\rm inst}^{(e\text{-ph})}(T)$, and $V_{\rm inst}^{(e\text{-}e)}(T)$ for all temperatures obtained numerically from Eq. (1) for the wire with $L/\xi_0 = 15$ are shown in Fig. 2. The universal behavior at small biases can be easily seen (inset). Since the ratio L/ξ_0 is not very large, the instability line becomes strongly dependent on the distribution function already for $V \gtrsim V_*$.

The most exciting feature of our results is the exponential growth of $V_{\rm inst}(T)$ with decreasing temperature in the noninteracting case, Eq. (11). Hence, even a small deviation of the distribution function from the two-step form (3) will drastically modify $V_{\rm inst}(T)$. As an example, consider the effect of a finite resistance of the normal terminals. Then the function $F_L(E)$ in Eq. (3) will be replaced by $F_L(E) = \beta F_0(E + eV/2) + (1 - \beta)F_0(E - eV/2)$, where V is the voltage applied to the NSN microbridge, and $\beta = R_T/(R_N + 2R_T)$ [R_T and R_N are the resistances of the N and S part of the junction, respectively]. The resulting $V_{\rm inst}(T)$ for $\beta = 0.1$ is shown by the dotted blue line in Fig. 2. While $V_{\rm inst}(T)$ is unchanged for small biases, it is strongly suppressed compared to $V^{\rm (free)}_{\rm inst}(T)$ for large biases.

V. LOW-TEMPERATURE BEHAVIOR

The exponential growth of $V^{\rm (free)}_{\rm inst}(T)$ in the noninteracting case formally implies that superconductivity at $T = 0$ might persist up to exponentially large voltages, $\ln[eV_{\rm inst}(0)/T_{c0}] \sim L/\xi_0 \gg 1$. This conclusion is wrong, since inelastic relaxation and heating become important with increasing V, even if they were negligible at $V = 0$. To study the low-T part of the instability line, we consider here a model of the e-ph interaction (e-e relaxation neglected) when the phonon temperature is assumed to coincide with the base temperature of the terminals and e-ph relaxation is weak at T_c: $l_{e\text{-ph}}(T_c) \gg L$ (as in Ref. 14).

With decreasing T below T_c, the instability line first follows Eq. (11). At the same time, $l_{e\text{-ph}}$ decreases and eventually the distribution function in the middle of the wire becomes nearly thermal with the effective temperature $T_{\rm eff}$. This happens when $T_{\rm eff}$ obtained from the heat balance equation,[20] $(eV/L)^2 \sim T^5_{\rm eff}/T^3_c l^2_{e\text{-ph}}(T_c)$, becomes so large that $l_{e\text{-ph}}(T_{\rm eff}) \sim L$. The corresponding voltage, $V_{\rm ph}$, can be estimated as $eV_{\rm ph}/T_c \sim [l_{e\text{-ph}}(T_c)/L]^{2/3}$. Consequently, the exponential growth (11) persists for voltages $V_* \lesssim V \lesssim V_{\rm ph}$, corresponding to the temperature range $T_{\rm ph} \lesssim T \lesssim T_*$, where with logarithmic accuracy $1 - T_{\rm ph}/T_{c0} \sim (\xi_0/L)^{2/3}$.

For higher biases, $V > V_{\rm ph}$, electrons in the central part of the wire have the temperature $T_{\rm eff}$. However, the parameter α, Eq. (8), is determined by the distribution function in the vicinity of the terminals which is not thermal. Matching the solution of the collisionless kinetic equation for $0 < x_L < l_{e\text{-ph}}(T_{\rm eff})$ at the effective right "boundary," $x_L = l_{e\text{-ph}}(T_{\rm eff})$, with the function (4) with $T(x) = T_{\rm eff}$, we obtain $\alpha \sim 1/l_{e\text{-ph}}(T_{\rm eff})$. Therefore, for $V \gtrsim V_{\rm ph}$ we get with logarithmic accuracy

$$\frac{eV_{\rm inst}(T)}{T_{c0}} \sim \frac{L}{\xi_0}\left(\frac{l_{e\text{-ph}}(T_c)}{\xi_0}\right)^{2/3}\left(\frac{T_{c0}-T}{T_{c0}}\right)^{5/2}. \quad (13)$$

Equation (13) corresponding to the case $a(V) \ll l_{e\text{-ph}} \ll L$ is different from the expression (10) when phonons are important already at T_c, and $l_{e\text{-ph}} \ll a(V) \ll L$. The scaling dependence of Eq. (13) on L indicates that the stability of the normal state is controlled by the applied current, similar to Eq. (10). At zero temperature, the instability current exceeds the thermodynamic depairing current by the factor of $[l_{e\text{-ph}}(T_c)/\xi_0]^{2/3} \gg 1$.

VI. DISCUSSION

Our general procedure locates the absolute instability line, $V_{\rm inst}(T)$, of the normal state for a voltage-biased NSN microbridge. Following experimental data,[14] we assumed that the onset of superconductivity is of the second order. While nonlinear terms in the TDGL equation are required to determine the order of the phase transition,[33] we note that were it of the first order, its position would be shifted to voltages higher than $V_{\rm inst}(T)$.

In the vicinity of T_c, the problem of finding $V_{\rm inst}(T)$ can be mapped onto a one-dimensional quantum mechanics in some potential $U(x)$. For small biases, $eV \ll T_{c0}$, the potential $U(x)$ does not depend on the distribution function details, explaining universality of the instability line, including the bifurcation from the single-mode to the bimodal superconducting state at $eV \sim 50E_{\rm Th}$ (Ref. 18) and nucleation of superconductivity in the vicinity of the terminals for larger biases.[10]

For $eV \gtrsim T_{c0}$, the potential $U(x)$ becomes a functional of the normal-state distribution function, producing $V_{\rm inst}(T)$ that is strongly sensitive to inelastic relaxation mechanisms in the wire. For the dominant e-ph interaction, the instability is controlled by the electric field $\mathcal{E} = V/L$ [Eqs. (10) and (13)], while in the opposite case [Eqs. (11) and (12)], the instability cannot be solely interpreted as current- or voltage-driven. At zero temperature, the (nonuniform) superconducting state can withstand a current which is parametrically larger than the thermodynamic depairing current.

The high sensitivity of $V_{\rm inst}(T)$ to the details of the distribution function opens avenues for its use as a probe of inelastic relaxation in the normal state. The shape of $V_{\rm inst}(T)$ can be further used to determine the dominating relaxation mechanism and extract the corresponding inelastic scattering rate.

ACKNOWLEDGMENTS

We are grateful to M. V. Feigel'man, A. Kamenev, T. M. Klapwijk, J. P. Pekola, V. V. Ryazanov, J. C. W. Song, and D. Y. Vodolazov for discussions.

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