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Mesoscopics in vortex core: level statistics and transport properties

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Abstract

Dissipation due to vortex motion in a strongly anisotropic type-II superconductor at low temperatures is studied. We focus on the moderately clean case of $\Delta^2/E_F \ll \tau^{-1} \ll \Delta$ and consider the low-velocity limit where dissipation is due to the Landau–Zener transitions. It is shown that the flux-flow $I(V)$ characteristic in the mixed state of a weakly coupled layered-superconductor can contain hysteretic branch, contrary to the cases of purely 2D and 3D superconductors.

Keywords: Landau–Zener transitions; Type-II superconductor; Vortex; Dissipation; *IV* characteristics

1. Introduction

It is known since the work of Caroli et al. [1] that in the presence of a vortex in a superconductor, the Bogolyubov–de Gennes equation possesses a set of localized quasiparticle levels with a small energy separation $\omega_0 \approx \Delta/(k_F \xi) \ll \Delta$. At relatively high temperatures $T \gg \omega_0$, it might be possible to neglect discreteness of these levels and think of the vortex core as of the region of a normal metal. Then, the Bardeen–Stephen phenomenological prediction [2] for the dissipative part of conductivity follows: $\sigma_{xx}^{(0)} \simeq \sigma_n(H_{c2}/B)$. Later, this result was verified microscopically in Refs. [3a,3b]. All these works were based on the quasiclassical approach that assumes the inelastic width of the core levels exceeding the mean level spacing ω_0.

Recently, an opposite situation when discreteness of the level structure becomes essential was considered [4–7]. These papers refer to strongly anisotropic (layered) superconductors that seemed to allow to neglect small interlayer coupling and treat the problem as 2D. In the superclean regime ($\omega_0\tau \gg 1$) considered in Ref. [5], the spectrum was found to be $2\omega_0$-periodic. The properties of the spectrum in the moderately clean regime ($\omega_0 \ll 1/\tau \ll \Delta$) were discussed in Refs. [7,8]. The result appears to be sensitive to the strength of a single

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impurity's potential. For strong impurities with large Born parameter $\theta \leq 1$, $2\omega_0$-rigidity of the spectrum persists up to $1/\tau \ll \theta^2\sqrt{\Delta\omega_0}$ [9], while for the model of a white noise disorder potential ($\theta \to 0$), the spectral statistics can be described [8] by an appropriate random matrix theory (RMT) [9].

Once spectral statistics is known, one can calculate the rate of energy dissipation for a moving vortex. When the realization of disorder potential in the core is changed, the levels move up and down randomly. For a small-enough velocity, the dissipation is due to rear Zener transitions between neighbouring levels. For impurities with large Born parameter, the conductivity in the moderately clean regime is $\sigma_{xx} \sim (\omega_0\tau)^{-2}\sigma_{xx}^{(0)}$ [7]. Within the RMT for a white noise potential, $\sigma_{xx} \approx \sigma^{(0)}$ and grows slowly with the increase of the electric field [4].

2. Discussion

In this work, we will show that weak *interlayer* coupling, completely neglected in the previous papers [4–7], may strongly affect σ_{xx}, and even lead to hysteretic $I(V)$ characteristics in the low-V region. We will consider the moderately clean regime in strongly anisotropic ($m_\mathrm{a} \ll m_\mathrm{c}$) superconductors with weak impurities, so that the RMT is applicable. Then, the energy is dissipated when the quasiparticle jumps from the n-th level in one layer to the m-th ($m \neq n$) level in the adjacent layer. For this process, levels $(1n)$ and $(2m)$ should meet at the same energy that is possible due to randomness of the level positions. Such crossings occur very often in the course of vortex motion since they are not prohibited by the level repulsion relevant for intralayer transitions. From the mathematical point of view, the system considered is equivalent to a set of independent random matrix ensembles for each layer with very small hopping between the adjacent ensembles. The crossing between the states $(1n)$ and $(2m)$ can be described by the Hamiltonian

$$H = \begin{pmatrix} E_{1n} + a_{1n}X & h_{1n,2m} \\ h^*_{1n,2m} & E_{2m} + a_{2m}X \end{pmatrix}, \tag{1}$$

where E_{pj} and a_{pj} are the position and velocity, respectively, of the j-th level from the p layer, X is the vortex coordinate, and $h_{1n,2m}$ is the interlayer tunneling matrix element. In a clean system, the states are labeled by the angular momentum μ and $h_{1\mu,2\mu'} = \mu\omega_0(m_\mathrm{a}/m_\mathrm{c})\delta_{\mu\mu'}$. In a dirty system, $\langle h_{1n,2m}\rangle = 0$, and the variance $\langle |h_{1n,2m}|^2\rangle = \omega_0^2(m_\mathrm{a}/m_\mathrm{c})^2\sum_\mu \mu^2\langle|\langle n|\mu\rangle|^2\rangle\langle|\langle m|\mu\rangle|^2\rangle$. The overlap between the exact state $|n\rangle$ and the pure state $|\mu\rangle$ can be evaluated as $\langle|\langle n|\mu\rangle|^2\rangle = (1/\pi) Im G_\epsilon^R(\mu)$, where $G_\epsilon^\mathrm{R}(\mu) = (\epsilon - \mu + ib)^{-1}$ is the retarded Green function, $\epsilon = n\omega_0$ is the average energy of the n-th layer and $b \simeq (\omega_0\tau)^{-1}\ \ln\Delta\tau$ is an effective number of levels mixed by disorder [8]. Due to the RMT spectral rigidity, the crossing is possible only for levels with $m - n \sim 1$. Then, the average square of the tunneling matrix element is given by

$$h_n^2 \equiv \langle |h_{1n,2m}|^2\rangle = \omega_0^2\left(\frac{m_\mathrm{a}}{m_\mathrm{c}}\right)^2 \frac{b^2+n^2}{2\pi b}. \tag{2}$$

Within the RMT, statistics of $h_{1n,2m}$ is Gaussian: $\mathscr{P}(h_{1n,2m}) = (1/(2\pi h_n^2))\exp(-(|h_{1n,2m}|^2/2h_n^2))$.

Near the crossing, the difference between the eigenvalues of the Hamiltonian (Eq. (1)) can be written as $\sqrt{(\Delta\epsilon)^2 + A^2(X-X_0)^2}$. To calculate the interlayer transition rate, one has to know the probability density $N_{mn}(\Delta\epsilon, A)$ to have an avoided crossing with given $\Delta\epsilon$ and A per unit displacement of the vortex. Following the method [10], we obtain

$$N_{mn}(\Delta\epsilon, A) = w_{mn}\frac{A}{\sqrt{2\pi}\,\sigma}\exp\left(-\frac{A^2}{8\sigma^2}\right)\frac{\Delta\epsilon}{4h_0^2}\exp\left(-\frac{(\Delta\epsilon)^2}{8h_0^2}\right). \tag{3}$$

According to this result, typical values of the level velocity, A, and the gap at the crossing, $\Delta\epsilon$, are of the order of $\sigma \equiv \omega_0 (n\omega_0\tau)^{1/2}$ [4] and h_n, respectively. In Eq. (3), $w_{mn} = \int_0^\infty \rho_m(E)\rho_n(E)\mathrm{d}E$; $\rho_n(E)$ is the probability density to find n-th level at the energy E.

To calculate the transition rate R_{mn}^{inter}, one should average the probability of the interlayer transition at a single crossing over the distribution (Eq. (3)). This results in

$$R_{mn}^{\text{inter}} = \begin{cases} 4\pi w_{mn} h_n^2, & \text{if } v \gg h_n^2/\sigma; \\ w_{mn} \dfrac{v^2\sigma^2}{\pi h_n^2}, & \text{if } v \ll h_n^2/\sigma. \end{cases} \tag{4}$$

Kinetic equation for the distribution function of quasiparticles is has form $((\partial f_n(t))/\partial t) + \Sigma_m R_{mn}(f_n - f_m) = \mathrm{St}_{\text{in}}$. Here, St_{in} is the inelastic relaxation collision integral, $R_{mn} = \delta_{|m-n|,1} R^{\text{intra}} + R_{mn}^{\text{inter}}$, where the intralevel transition rate is given by $R^{\text{intra}} = \pi n \tau v^2$ [4]. The dependence of R_{mn}^{inter} on the difference $m-n$ is due to the factor w_{mn}. It is a short-range, nearly symmetric function of $m-n$, and has the order of the inverse level spacing, ω_0^{-1}. Then, the kinetic equation can be reduced to the local form: $((\partial f_n(t))/\partial t) + R_n((\partial^2 f_n)/\partial n^2) = \mathrm{St}_{\text{in}}$. An effective spectral diffusion coefficient $R_n = R^{\text{intra}} + (1/2)\Sigma_{m=1}^{\infty}(m-n)^2 R_{mn}^{\text{inter}}$. The latter term contains the sum $W_n = \Sigma_{m=1}^{\infty}(m-n)^2 w_{mn}$, which is a very slow (logarithmic) function of n and is always of the order of ω_0^{-1} for experimentally available level numbers.

According to Eqs. (2) and (4), R_n is a complicated function of the level number n and the vortex velocity v. Were we to consider n-independent transition rates $R_n = R$, the result for the rate W of the energy dissipation would be independent of St_{in} and given by $\omega_0 R$ [10]. In the present case, knowledge of the inelastic interaction is essential. The general form of the collision integral is

$$\mathrm{St}_{\text{in}} = \sum_{m>n} \Gamma_{\text{in}}[(m-n)\omega_0] f_m[1-f_n] - \sum_{m<n} \Gamma_{\text{in}}[(m-n)\omega_0] f_n[1-f_m], \tag{5}$$

where $\Gamma_{\text{in}}(\omega)$ is the phonon relaxation rate with energy transfer ω, and $T=0$ case is implied. Phenomenologically, one can assume a power-low dependence $\Gamma_{\text{in}}(\omega) = \omega_0 \gamma_p (\omega/\omega_0)^p$. According to estimate of Ref. [4] for the case of electron–phonon interaction, the relaxation exponent $p=1$ and $\gamma_1 \simeq (\omega_0\tau)\omega_0^2 n/\rho s^3$, where ρ is the mass density of the crystal and s is the sound velocity. Below we will consider the case of an arbitrary value of p.

The master equation cannot be solved analytically for an arbitrary behavior of R_n. However, in the limiting cases, the method of dimensional estimates can be used. Such an analysis shows that there are three universal regimes depending on the strength of the phonon relaxation. It appears to be convenient to measure it by the dimensionless parameter $\kappa = (b/N_0)^{p+3}$, where $N_0 = (R^{\text{intra}}/\omega_0\gamma_p)^{1/(p+3)}$ is the width of the stationary distribution function on the border of applicability of the quasiclassical approach, at $v \sim v_{\text{K}} = (\Delta/p_{\text{F}})/\sqrt{k_{\text{F}} l}$ [4]. Introducing the dimensionless velocity $u = v/v_{\text{K}}$, and hopping $t = h_0/\omega_0 \ll 1$, we present the final result for the dissipation rate $W(v)$ normalized to the Bardeen–Stephen dissipation rate $W^{(0)} = \eta_0 v^2 = \pi\hbar n \omega_0 \tau v_{\text{K}}^2 u^2$ in the three regimes:

$$\text{For } t^2 \ll \kappa: \quad \frac{W}{W^{(0)}} = \begin{cases} t^{-2}, & \text{if } u \ll t^2; \\ u^{-2} t^2, & \text{if } t^2 \ll u \ll t; \\ 1, & \text{if } t \ll u. \end{cases} \tag{6}$$

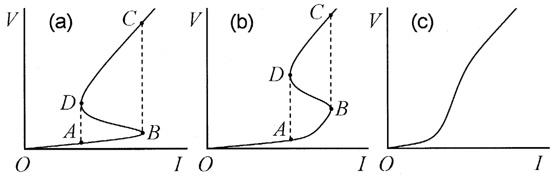

Fig. 1. Schematic view of the mixed-state $I(V)$ characteristics for the cases of strong (a), intermediate (b) and weak (c) phonon relaxation. In the cases (a) and (b) hysteretic $I(V)$s are predicted: upon increase and then decrease of a current, the voltage follows the loop A-B-D-C-A.

$$\text{For } t^{p+3} \ll \kappa \ll t^2: \quad \frac{W}{W^{(0)}} = \begin{cases} t^{-2}, & \text{if } u \ll t\kappa^{1/2}; \\ u^{-\frac{4}{p+5}} t^{-\frac{2(p+3)}{p+5}} \kappa^{\frac{2}{p+5}}, & \text{if } t\kappa^{1/2} \ll u \ll t^{\frac{2(p+3)}{p+1}} \kappa^{-\frac{2}{p+1}}; \\ u^{-2} t^{\frac{2(p+3)}{p+1}} \kappa^{\frac{2}{p+1}}, & \text{if } t^{\frac{2(p+3)}{p+1}} \kappa^{-\frac{2}{p+1}} \ll u \ll t^{\frac{p+3}{p+1}} \kappa^{-\frac{1}{p+1}}; \\ 1, & \text{if } t^{\frac{p+3}{p+1}} \kappa^{-\frac{1}{p+1}} \ll u. \end{cases} \tag{7}$$

$$\text{For } \kappa \ll t^{p+3}: \quad \frac{W}{W^{(0)}} = \begin{cases} t^{-2}, & \text{if } u \ll t\kappa^{1/2}; \\ u^{-\frac{4}{p+5}} t^{-\frac{2(p+3)}{p+5}} \kappa^{\frac{2}{p+5}}, & \text{if } t\kappa^{1/2} \ll u \ll t^{-\frac{p+3}{2}} \kappa^{-\frac{1}{2}}; \\ 1, & \text{if } t^{-\frac{p+3}{2}} \kappa^{-\frac{1}{2}} \ll u. \end{cases} \tag{8}$$

I–V characteristics in the mixed state are determined from Eqs. (6)–(8) by the standard relations for the dissipative power $W(v) = v(\Phi_0/c)j$ and electric field $E = (v/c)B$. Schematically, $I(V)$s are shown in Fig. 1. The most interesting are the cases (a) and (b) where formal existence of an intermediate unstable branch (between points B and D in Fig. 1) with $W(v) = \text{const}(v)$, that is $E \propto j^{-1}$, leads actually to the hysteretic shape of the DC current–voltage characteristic.

In our consideration, we assumed that the interlayer hopping is small compared to ω_0. It is valid as long as the width of the stationary distribution function is less that the mass anisotropy m_c/m_a. We have also ignored pinning, assuming that either the vortex lattice is rigid or the frequency exceeds the pinning frequency.

3. Conclusion

To conclude, we considered the flux flow dynamics in the moderately clean case in layered s-wave superconductors with weak impurities. We have shown that weak interlayer quasiparticle transitions within the vortex core strongly modify vortex friction in the range of small vortex velocities, compared with, both 3D semiclassical [3a,3b,4] and purely 2D mesoscopic [4] predictions. As a result, a pronounced hysteresis in the I–V curve appears. Our theory can probably be relevant for the layered superconductor $NbSe_2$.

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References

[1] C. Caroli, P.G. De Gennes, J. Matricon, Phys. Lett. 9 (1964) 307.
[2] J. Bardeen, M.J. Stephen, Phys. Rev. 140 (1965) 1197A.
[3a] L.P. Gor'kov, N.B. Kopnin, JETP Lett. 38 (1973) 195.
[3b] A.I. Larkin, Yu.N. Ovchinnikov, JETP Lett. 23 (1976) 187.
[4] M.V. Feigel'man, M.A. Skvortsov, Phys. Rev. Lett. 78 (1997) 2640.
[5] A.I. Larkin, Yu.N. Ovchinnikov, Phys. Rev. B 57 (1998) 5457.
[6] A.A. Koulakov, A.I. Larkin, Phys. Rev. B 59 (1999) 1383.
[7] A.A. Koulakov, A.I. Larkin, cond-mat/9810125, preprint.
[8] M.A. Skvortsov, V.E. Kravtsov, M.V. Feigel'man, JETP Lett. 68 (1998) 84.
[9] A. Altland, M.R. Zirnbauer, Phys. Rev. B 55 (1997) 1142.
[10] M. Wilkinson, J. Phys. A: Math. Gen. 21 (1988) 4021.