Level statistics inside the core of a superconductive vortex

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A microscopic theory of the Efetov supermatrix sigma-model type is constructed for the low-lying electron states in a mixed superconductive–normal system with disorder. This technique is used for the study of the localized states in the core of a vortex in a moderately clean superconductor with $\tau^{-1}\gg\omega_0\sim\Delta^2/E_F$. At low energies $\epsilon\ll\omega_{\mathrm{Th}}\sim(\omega_0/\tau)^{1/2}$, the energy level statistics is described by the ''zero-dimensional'' limit of this supermatrix theory, and the result for the density of states is equivalent to that obtained within Altland–Zirnbauer random matrix model. Nonzero modes of the sigma model increase the mean interlevel distance by the relative amount $[2\,\ln(1/\omega_0\tau)]^{-1}$.
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There is a great deal of activity directed to the study of electron energy levels and wave functions in disordered normal metals,[1] where they govern the low-temperature transport properties. In (s-wave) superconductors, disorder is usually of less importance, since the excitation spectrum has a nonzero gap and single-electron states are almost empty at $T\ll\Delta$. The situation is quite different in mixed superconductive–normal systems (for a recent review see Ref. 2) where the gap in the excitation spectrum can be: i) very low compared to the bulk value Δ, or ii) exactly zero. An example of the first case is presented by an S–N–S sandwich with the thickness of the N region $L_{\mathrm{N}}\gg\xi,l$. At generic values of the phase difference φ between superconductors the gap in the electron spectrum in the N region is of the order of the Thouless energy $E_{\mathrm{Th}}=D/L_{\mathrm{N}}^2\ll\Delta$. To calculate the density of states (DoS) fluctuations at $\epsilon>E_{\mathrm{Th}}$, and other mesoscopic effects in such systems, a field theory was developed[3] which is an extended version of Efetov's supermatrix σ model. A qualitatively different situation arises in case ii), which is realized, e.g., in the same S–N–S sandwich at $\varphi=\pi$ (Ref. 4) or in a variety of situations where an external magnetic field is present. Now the DoS is nonzero at arbitrary low energies, and quantum interference due to Andreev scattering strongly affects even the average DoS $\langle\rho(\epsilon)\rangle$. General approach to this kind of systems was initiated by Altland

and Zirnbauer (AZ),[5] who employed a generalized random-matrix (RM) approach. The particle–hole symmetry of the Bogolyubov–de Gennes (BdG) Hamiltonian leads to the constraints to be imposed on the RM Hamiltonians. The precise form of the constraint depends on the presence or absence of time inversion and spin rotation symmetries. Thus AZ identified 4 new classes of RM ensembles appropriate for the description of this kind of S–N–S systems. Crossover between such classes has been considered in Ref. 6 using the space-independent supermatrix sigma model. While the AZ approach is highly suggestive, it has the same limitation as any *ad hoc* RM theory, i.e., the limits of its applicability to some real physical system are left undetermined.

In the present letter we develop a microscopic field-theory approach to an example of a system of type ii), namely, to the core of a superconductive vortex. It has been known since the paper by Caroli, de Gennes, and Matricon (CdGM)[7] that the BdG equations near the vortex possess localized solutions with energies well below the bulk value Δ. The spacing ω_0 between these localized levels is of the order of $\Delta/(k_F\xi)$ and disappears in the quasiclassical limit $k_F\xi\rightarrow\infty$. Thus it was tempting to consider the vortex core as a kind of a ''normal tube'' inside a superconductor, and in many cases such a simplified picture was found[8] to be at least qualitatively correct. Later it was demonstrated[9] that the presence of a quasi-continuum spectrum branch localized on the vortex follows from general topological arguments. However, it is not always possible to consider the chiral branch as a continuous one. It was shown recently[10,11] that the discreteness of the localized energy levels becomes of real importance in layered superconductors at sufficiently low temperatures. In the previous paper[10] we employed the AZ approach to find low-current nonlinearities in the current–voltage relation in a mixed state of a moderately clean superconductor (the mean free path $l\gg\xi$, but $l\ll\xi(k_F\xi)$). In such a case the inverse elastic scattering time $1/\tau$ is much larger than interlevel spacing ω_0, and therefore the applicability of an appropriate RM model (which is, in fact, class C of the AZ classification) seems natural. Another, qualitatively different, limiting case of an ultraclean superconductor with extremely low concentration of impurities ($l\gg k_F\xi^2$) was considered recently by Larkin and co-workers.[11] In the present letter we again consider a moderately clean limit $\omega_0\ll 1/\tau\ll\Delta$, now within a microscopic approach starting from the BdG equations in the presence of a Gaussian random potential. We derive the conditions under which the AZ class C statistics is indeed realized in the vortex core, and estimate the scale of nonuniversal corrections to it. We consider here a purely 2D superconductor, which is a good approximation for the case of a strong layered anisotropy (see Ref. 10 for more details).

Below we briefly present our method and results (see Ref. 12 for details). In the present problem even the calculation of the average single-particle quantities is not trivial and cannot be done within the quasiclassical theory, as long as low energies $\epsilon\sim\omega_0$ are considered. Thus our goal is to derive a field-theory technique for the calculation of the average DoS. To average the Green function over disorder, we use a standard trick[1] of representing it as the functional integral over both the Grassmann (χ) and usual complex (S) fields which combine into the superfield Φ. The most direct way would be to work with a real-space-dependent superfield $\Phi(\mathbf{r})$; in this way we would obtain a field theory in terms of a supermatrix $Q(\mathbf{r})$. On the other hand, low-lying states of the chiral branch depend upon a single quantum number only, as well as for a generic 1D problem. Therefore, in the basis of such states the BdG Hamiltonian can be represented as a random $N\times N$ Hermitian matrix (where $N\sim\Delta/\omega_0$ is the total number of localized states in the core) of a certain structure and symmetry which we will discuss below. In the clean limit, $1/\tau\ll\Delta$, the admixture of delocalized ($\epsilon>\Delta$) states to the low-lying ones can be neglected. Thus it is convenient first to reduce the full 2D problem to a sort of RM problem that can be further reduced to a 1D field theory explicitly containing the chiral spectrum branch only.

In the basis of the CdGM states $\Psi_\mu(\mathbf{r})=A(J_{\mu-1/2}(k_Fr),J_{\mu+1/2}(k_Fr))^T e^{i\mu\theta}e^{-K(r)}$ determined in Ref. 7 (here $A\sim\sqrt{k_F/\xi}$ is the normalization constant, θ is the azimuthal angle in the real space, $\mu\in[-N/2,N/2]$ is the angular momentum that takes half-integer values, and $K(r)=(1/\hbar v_F)\int_0^r\Delta(r')dr')$, the full Hamiltonian takes the form $\langle\mu|\hat{H}|\mu'\rangle=\omega_0\mu\delta_{\mu,\mu'}+\langle\mu|\hat{V}|\mu'\rangle$, where the second term is due to the random white-noise impurity potential $U(\mathbf{r})$ with the variance $\langle U(\mathbf{r})U(\mathbf{r}')\rangle=\delta(\mathbf{r}-\mathbf{r}')/(2\pi\nu\tau)$; correspondingly, in the functional integral one should use a μ-dependent supervector Φ_μ instead of the superfield $\Phi(\mathbf{r})$. This Hamiltonian has the symmetry

$$\hat{H}=-\hat{\gamma}\hat{H}^T\hat{\gamma}^T;\quad \langle\mu|\hat{\gamma}|\mu'\rangle=(-1)^{\mu+1/2}\delta_{\mu+\mu'}, \tag{1}$$

which follows from an identity $\Psi_{-\mu}(\mathbf{r})=(-1)^{\mu+1/2}i\tau_y\Psi^*_\mu(\mathbf{r})$ that reflects the basic symmetry property of the Hamiltonian; τ_y is the Pauli matrix in the Nambu space.

The standard way to solve a complicated random matrix problem is to represent it in a form of the effective field theory. In order to reduce the RM problem given by Eq. (1) to the 1D field theory we make a continuous Fourier transform (considering N as very large) from the momentum variable μ to the ''angle'' $\phi\in[0,2\pi)$, so our superfield will be defined as $\Phi(\phi)=\Sigma_\mu\Phi_\mu e^{-i(\mu-1/2)\phi}\equiv\Phi_\phi$. Now we can write down an expression for the ''partition function'' ($\epsilon_+\equiv\epsilon+i\delta$):

$$Z^R(\epsilon)=\int\exp i\int\frac{d\phi}{2\pi}\left\{\Phi^*_\phi\left(\epsilon_+-i\omega_0\frac{\partial}{\partial\phi}-\frac{\omega_0}{2}\right)\Phi_\phi\right.$$
$$\left.-\int\frac{d\phi'}{2\pi}\Phi^*_\phi V(\phi,\phi')\Phi_{\phi'}\right\}D^2\Phi_\phi. \tag{2}$$

The matrix elements $V(\phi,\phi')$ of the random potential in the ϕ space obey the symmetry relationship that follows from Eq. (1) and are given by $V(\phi,\phi')=-e^{i(\phi-\phi')}V^*(\phi+\pi,\phi'+\pi)=A^2\int d^2\mathbf{r}w_{\phi\phi'}(r,\theta)U(\mathbf{r})e^{-2K(r)}$. The function $w_{\phi\phi'}$ can be written, using the Bessel function summation formulas, as

$$w_{\phi\phi'}(r,\theta)=(1-e^{i(\phi-\phi')})\exp\{-ik_Fr[(\sin\phi-\sin\phi')$$
$$\times\cos\theta+(\cos\phi-\cos\phi')\sin\theta]\}. \tag{3}$$

All features of the theory are encoded in the pair correlator $\mathcal{W}(\phi_1,\phi_2,\phi_3,\phi_4)=\langle V(\phi_1,\phi_2)V(\phi_3,\phi_4)\rangle$ where the averaging is performed over the Gaussian distribution of the random potential $U(\mathbf{r})$. Since the typical value of $k_Fr\sim k_F\xi\gg 1$, the correlator $\mathcal{W}(\phi_1,\phi_2,\phi_3,\phi_4)$ is appreciably nonzero only when the oscillating exponents in Eq. (3) nearly cancel each other, i.e., when its arguments ϕ_i are pairwise coinciding:[12]

$$\mathcal{W}(\phi_1,\phi_2,\phi_3,\phi_4)=\frac{g\omega_0^2}{\pi}T(\phi_{12}+\pi)(2\pi)^2[\delta(\phi_{14})\delta(\phi_{23})$$
$$-e^{i\phi_{12}}\delta(\phi_{13}+\pi)\delta(\phi_{24}+\pi)], \tag{4}$$

where $\phi_{kl}\equiv\phi_k-\phi_l$, $g=2A^4/\pi\nu\tau\omega_0^2k_F^2\sim 1/\omega_0\tau\gg 1$ and the kernel T is given by

$$T(\phi)=\frac{\pi}{2}\left|\cot\frac{\phi}{2}\right|,\quad \text{if } |\phi|>\frac{1}{\sqrt{N}};\quad T(\phi)\approx\sqrt{N},\quad \text{if } |\phi|<\frac{1}{\sqrt{N}}. \tag{5}$$

The δ-function approximation (4) for the correlator $\mathcal{W}$ is valid as long as the scale of the angular variations of the field $\Phi(\phi)$ (below it will be seen to be $\ell=[g\ln(N/g)]^{-1}$) is longer than the actual[12] width $w(\phi_{12})\sim|N\sin(\phi_{12})|^{-1}$ of those δ functions. Thus the following derivation is strictly valid under the condition $w(\ell)\ll\ell$, which is equivalent to

$$\tau\sqrt{\omega_0\Delta}\gg\ln\,\Delta\tau. \tag{6}$$

Below we will assume that the inequality (6) is fulfilled.

The next step of the σ model derivation is to average the partition function (2) using Eqs. (4) and (5). Before doing that we need to take explicitly into account the symmetry (1), which amounts to a doubling of the number of components of the supervector $\Phi(\phi)$. Thus we introduce (cf. with a similar procedure in Ref. 6) an additional 2×2 ''particle–hole'' (PH) space and define a 4-dimensional superfield $\psi(\phi)=2^{-1/2}(\Phi(\phi),e^{i\phi}\Phi^*(\phi+\pi))^T$. Next we define the bar-conjugate superfield as $\overline{\psi}(\phi)=\psi^+\sigma_z=[C(\phi)\psi(\phi+\pi)]^T$, with $C(\phi)=-e^{-i\phi}\sigma_zC_0$, where σ_z is the Pauli matrix in the PH space, and the 4×4 matrix C_0 consists of the blocks $C_0^{pp}=C_0^{hh}=0$, $C_0^{ph}=1$, and $C_0^{hp}=k$, where $k=\mathrm{diag}(1,-1)$ acting in the Fermi–Bose space. After an averaging over the disorder, the effective action $\mathcal{A}\{\psi\}$ for the retarded Green function $\mathcal{G}^R(\epsilon)=-i\int\Phi^f(\Phi^f)^+\exp[\mathcal{A}\{\psi\}]D\psi^*D\psi$ (where Φ^f means the fermionic component of Φ) can be written as (we denote $\psi_j\equiv\psi(\phi_j)$):

$$\mathcal{A}=i\int\frac{d\phi_1}{2\pi}\overline{\psi}_1\left(\epsilon_+\sigma_z-i\omega_0\frac{\partial}{\partial\phi_1}-\frac{\omega_0}{2}\right)\psi_1-\frac{g\omega_0^2}{\pi}$$
$$\times\int\int\frac{d\phi_1d\phi_2}{(2\pi)^2}T(\phi_{12}+\pi)\overline{\psi}_1\psi_2\overline{\psi}_2\psi_1. \tag{7}$$

The second term in the action (7) is similar to that of the 1D tight-binding model with off-diagonal random matrix elements with variance decaying as $1/|x|$, as long as we are interested in scales $|x|\equiv|\phi_1-\phi_2+\pi|\ll\pi$. Thus the usual 1D localization is absent in our problem because of the long-range nature of the off-diagonal disorder (cf. Ref. 13).

There is also another way of considering this term, which helps to gain some intuition about its effect. Namely, one can think of the variable ϕ as an angle associated with the 2D quasiparticle momentum $p=k_F\{\cos\phi,\sin\phi\}$. Then the last term in Eq. (7) corresponds to a 2D particle–hole scattering strongly enhanced in the forward direction. For such a singular scattering one has to define two scattering lengths ℓ and $\ell_{tr}\gg\ell$ (cf. with a similar situation discussed in Ref. 14): $1/\ell\propto g\int d\phi\ \sigma(\phi)=g\ \ln(N/g)$, and $1/\ell_{tr}\propto g\int d\phi\ \sigma(\phi)(1-\cos\phi)=g\gg 1$, where $\sigma(\phi)$ is the differential cross section and $\phi=\phi_1-\phi_2+\pi$. For careful evaluation of the logarithmically divergent scattering rate

$1/\ell$, one should use the self-consistent Born approximation (SCBA), which takes into account both terms in Eq. (4). It is equivalent to taking into account the ''noncrossing'' diagrams that can be generated by a perturbative expansion of $\exp[\mathcal{A}\{\psi\}]$ in powers of g. For $\epsilon/\omega_0 \ll 1/\ell$, the ''crossing diagrams'' of the same order in g turn out to be small by the parameter $\ell/\ell_{tr} = 1/\ln(N/g)$. It stands for the usual quasiclassical parameter $(k_F \ell_{tr})^{1-d}$ in this effectively 1D problem.

The existence of the small parameter $1/\ln(N/g) = 1/\ln \Delta\tau$ that allows one to neglect the ''crossing diagrams'' implies that one can derive an effective field theory (nonlinear sigma model) which describes the low-energy behavior of the averaged Green function $G^R(\epsilon)$ for $\epsilon/\omega_0 \ll 1/\ell = g\ \ln(N/g)$. This can be done in a standard way[1] by the Hubbard–Stratonovich decoupling of the quartic term in Eq. (7) and a further saddle point approximation controlled by the parameter $1/\ln(N/g)$. Because of the symmetry relation (1) and the corresponding relation between $\overline{\psi}$ and ψ, one has to perform both a local decoupling containing $P(\phi)\psi(\phi)\otimes\overline{\psi}(\phi)$ and a nonlocal one containing $R(\phi_1,\phi_2)\psi(\phi_1)\otimes\psi^T(\phi_2)$. Under the condition $\ell \gg 1/\sqrt{N}$ given by Eq. (6), both decouplings are important in order to obtain the correct form for the imaginary part of the Green function in the saddle-point approximation $P(\phi) = P_0$, $R(\phi_1,\phi_2) = R_0(\phi_1 - \phi_2)$, which is equivalent to the SCBA:

$$G_\epsilon(\phi) = -\frac{2\pi i}{\omega_0}\sigma_z \theta(-\sigma_z\phi) e^{-|\phi|/\ell} e^{-i\frac{\epsilon}{\omega_0}\phi}, \quad P_0 = \frac{T_0}{\omega_0}\sigma_z, \quad R_0(\phi) = \frac{i}{\pi} T(\phi) G_\epsilon(-\phi), \tag{8}$$

where $T_0 = \int_0^{2\pi} T(\phi)\, d\phi/2\pi \approx \frac{1}{2}\ln N$. In general, the mth Fourier harmonic of the kernel $T(\phi)$ is given by $T_m \approx \ln(\sqrt{N}/|m|)$ for $1 \ll m \ll \sqrt{N}$.

Mesoscopic fluctuations are known[1] to be described by the slow rotations of the saddle-point solution, which are represented in our case as $P(\phi) = U^{-1}(\phi) P_0 U(\phi)$, $R(\phi,\phi') = U^{-1}(\phi) R_0(\phi - \phi') U(\phi')$. The corresponding action that describes the low-energy spectral properties of the CdGM levels, reads:

$$\begin{aligned}\mathcal{A}_\sigma[Q,U] = &-\frac{\pi g}{4} T_0^2 \int\int \frac{d\phi_1 d\phi_2}{(2\pi)^2} T^{-1}(\phi_1 - \phi_2 + \pi)\,\mathrm{Str}\, Q(\phi_1) Q(\phi_2) \\ &- \frac{\pi i}{2}\int \frac{d\phi}{2\pi}\mathrm{Str}\left(\frac{\epsilon}{\omega_0}\sigma_z Q(\phi) - i\sigma_z U(\phi)\frac{\partial U^{-1}(\phi)}{\partial\phi}\right),\end{aligned} \tag{9}$$

where $Q(\phi) = U^{-1}(\phi)\sigma_z U(\phi)$, and $U(\phi)$ is a π-periodic, pseudo-unitary ($U^{-1}(\phi) = \overline{U}(\phi)$) matrix. The action (9) is valid for energies $\epsilon \ll \omega_0/\ell = \tau^{-1}\ln \Delta\tau$.

The supermatrix Q can be represented in the form $Q(\phi) = \sigma_z[1 + W(\phi) + \frac{1}{2}W^2(\phi) + O(W^3)]$, with the supermatrix W being purely off-diagonal in the PH space. Then the symmetry $Q = \overline{Q}$ and convergence arguments lead to the following form for the W_{ph} and W_{hp} blocks:

$$W_{ph}(\phi) = \begin{pmatrix} iz(\phi) & \alpha_1(\phi) \\ \alpha_1(\phi) & 0 \end{pmatrix}_{fb}, \quad W_{hp}(\phi) = \begin{pmatrix} iz^*(\phi) & \alpha_2(\phi) \\ -\alpha_2(\phi) & 0 \end{pmatrix}_{fb}. \tag{10}$$

Here z is a complex number and α_i are the Grassmann numbers. Expanding over $W(\phi)$, we obtain in the quadratic approximation

$$\mathcal{A}_2[W_m]=\frac{\pi}{4}\,\mathrm{Str}\sum_m\left\{2g\left(\sum_{k=0}^{|m|-1}\frac{1}{2k+1}\right)+i\left(m\sigma_z-\frac{\epsilon}{\omega_0}\right)\right\}W_{2m}W_{-2m}, \tag{11}$$

where the W_m are the mth harmonics of the field $W(\phi)$. Note that in Eq. (11) only even harmonics enter; odd harmonics, as well as the ''longitudinal'' modes, have a larger gap of the order of ω_0/ℓ and are excluded from the sigma-model action.

Equation (11) sets a characteristic scale $L=(g\ \ln g)^{-1}$ for the angular variations of the matrices $U(\phi)$. This scale should be larger than the scattering length ℓ. Only in that case can one neglect higher terms of the gradient expansion in powers of $\partial U/\partial\phi$, as was done in deriving Eq. (9). Comparing to $\ell=[g\ \ln(N/g)]^{-1}$ we see that the parameter of the gradient expansion, $\ell/L=\ln g/\ln(N/g)$, is small if the condition (6) is fulfilled. The length L determines the angular size of the elementary propagator corresponding to the sigma model (9). In this respect it is analogous to the system size in the usual weak-localization problem. The fact that $\ell/L\ll 1$ in our problem tells us that the problem is essentially not ballistic, though it is not diffusional either, since $\ell_{tr}/L=\ln g\gg 1$.

An important property of the action (9) is that it takes a universal form if U is independent of ϕ. At low energies the main contribution comes from the zeroth harmonics of $Q(\phi)$, i.e., the problem reduces to the zero-dimensional σ model. The uniform supermatrix Q is parametrized by 2 real variables (one of which appears to be cyclic) and 2 Grassmann variables, so the final expression for the average DoS is

$$\begin{aligned}\langle\rho(\epsilon)\rangle&=\frac{1}{4\tilde{\omega}_0}\Re\int_0^{\pi}d\theta\int d\eta d\zeta\,\frac{\sin\theta}{1-\cos\theta}[(1+\cos\theta)\\&\quad+2\eta\zeta(1-\cos\theta)]e^{\pi i\frac{\epsilon}{\tilde{\omega}_0}(1-\cos\theta)}\\&=\frac{1}{\tilde{\omega}_0}\left(1-\frac{\sin(2\pi\epsilon/\tilde{\omega}_0)}{2\pi\epsilon/\tilde{\omega}_0}\right).\end{aligned} \tag{12}$$

The functional form of this result coincides with the result of the AZ phenomenological approach.[5] However, it is expressed via the renormalized mean level spacing $\tilde{\omega}_0=\omega_0(1+(1/2\ln g))$. The renormalization is due to the contributions of higher $W_{m\geq 2}$ modes, which lead[12] to the decrease of the DoS for $\epsilon\leq\omega_0/L=g\omega_0\ln g$ by the relative amount of $\delta\omega_0/\omega_0=1/(2\ \ln g)\ll 1$. At higher energies this correction decreases as $\delta\omega_0/\omega_0\propto(g\omega_0\ln g)^2/\epsilon^2$. This correction can be found using a general approach,[15] in which the perturbative treatment of the nonzero modes leads to the ''induced'' terms in the 0D action. It is given by the single-cooperon diagram, which is absent in the usual normal-metal problems,[1,15] from the formal point of view, the difference stems from the absence of the BB block in the parametrization (10). The usual[15] two-cooperon diagram leads to the ''induced'' term $\propto(\epsilon/\omega_0)^2(g\ \ln g)^{-1}$ (cf. Ref. 3). The possibility of neglecting this term determines the upper limit of energies where a purely 0D description is valid: $\epsilon\leq\omega_{\mathrm{Th}}=\omega_0\sqrt{g\ \ln g}$.

To conclude, we have derived microscopically the supersymmetric field theory for the statistics of the localized electron levels inside the vortex in a moderately clean superconductor. Our supermatrix σ model, Eq. (9) was derived in leading order in the quasiclassical parameter $1/\ln \Delta\tau$. The approach proposed previously in Ref. 5 is shown to be valid in the low-energy range $\epsilon \leqslant \omega_{\mathrm{Th}} = [(\omega_0/\tau)\ln(1/\omega_0\tau)]^{1/2}$, where the 0D σ model is applicable. Mixing between zero- and higher modes leads to a decrease of the DoS by a relative amount $[2\ln(1/\omega_0\tau)]^{-1}$ at energies $\epsilon \leqslant \tau^{-1}\ln(1/\omega_0\tau)$.

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